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Article

Simultaneous observation of quantum contextuality and quantum nonlocality

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ABSTRACT

Quantum nonlocality and quantum contextuality are the most curious properties that change our understanding of nature, and were observed independently in recent decades. One important question is whether both properties can be observed simultaneously. In this paper, we show that in a qutrit-qutrit system we can observe quantum nonlocality and quantum contextuality at the same time. From the perspective of quantum information, our experiment proves in principle that the two resources, quantum nonlocality and quantum contextuality, can be utilized simultaneously.

1. Introduction

Quantum mechanics (QM) has radically subverted our intuition of nature. The most striking aspects of it is quantum nonlocality which is revealed by Bell theorem [1] and quantum contextuality which is revealed by Bell-Kochen-Specker theorem [2,3]. It has been recently recognized that quantum nonlocality is essential for device-independent secure communication [4–6], and quantum contextuality supplies the power for universal quantum computation via magic state distillation [7–11].

The relation between quantum contextuality and quantum nonlocality is a fundamental problem in QM. Stairs [12], Heywood and Readhead [13] found that single-particle contextuality can be transferred to two-particle nonlocality in the presence of Einstein-Podolsky-Rosen (EPR) correlations between two particles of spin 1 or higher. In a recent work, Cabello [14] claimed that contextuality played a fundamental role in quantum nonlocality. This means that contextuality may play a more important role in QM, and this phenomenon has been observed in experiment [15]. It is very surprising that if one restricts oneself to simple forms of nonlocality, such as Clauser-Horne-Shimony-Holt (CHSH) inequality [16], and single-particle contextuality, such as Klyachko-Can-Binicioglu-Shumovsky (KCBS) inequality [17], there are monogamies between them [18–21]. So an important question is arisen, "Can we have single-particle contextuality and two-particle nonlocality in the same system?"

In this paper, we show that in our scheme one can observe quantum contextuality and quantum nonlocality simultaneously in a qutrit-qutrit system. Alice and Bob shares two maximally entangled qutrits, and they can observe a violation of a nonlocal Bell inequality [22]. At the same time, Alice can observe a violation of local BKS inequality [22] using the same observables that used to disprove the nonlocal Bell inequality. This sheds new light on the old problem: what is the relation between contextuality and nonlocality, and whether both resources can be observed simultaneously in an experiment.

2. Theory

Suppose the two qutrits are initially prepared on a maximally entangled state

$$|\Psi\rangle = (|00\rangle + |11\rangle + |22\rangle)/\sqrt{3}. \quad (1)$$

The measurements can be defined as $A_i = B_i = I - 2|a_i\rangle\langle|a_i|$ $(i = 1, 2, \ldots, 13)$ in which I is a 3×3 identity matrix and $|a_i\rangle$ are the following three-dimensional unit vectors

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$$|a_1\rangle=\frac{1}{\sqrt{3}}\begin{pmatrix}-1\\1\\1\end{pmatrix},\quad |a_{5,6}\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\\pm1\end{pmatrix},\quad |a_{11}\rangle=\begin{pmatrix}1\\0\\0\end{pmatrix},$$
$$|a_2\rangle=\frac{1}{\sqrt{3}}\begin{pmatrix}1\\-1\\1\end{pmatrix},\quad |a_{7,8}\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\\pm1\end{pmatrix},\quad |a_{12}\rangle=\begin{pmatrix}0\\1\\0\end{pmatrix},$$
$$|a_3\rangle=\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\-1\end{pmatrix},\quad |a_{9,10}\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\\pm1\\0\end{pmatrix},\quad |a_{13}\rangle=\begin{pmatrix}0\\0\\1\end{pmatrix},$$
$$|a_4\rangle=\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}. \tag{2}$$

Any local hidden variable theories (LHVTs) satisfies [22]

$$\begin{aligned}\langle\beta\rangle\equiv\frac{1}{2}\Bigg(&\sum_{i=1}^{4}\langle A_i\rangle-\sum_{i=1}^{4}\sum_{k=5}^{10}\Gamma_{ik}\langle A_iB_k\rangle+\sum_{i=5}^{10}\langle A_iB_i\rangle-\sum_{i=5}^{10}\sum_{k=5}^{10}\Gamma_{ik}\langle A_iB_k\rangle\\&-\sum_{i=11}^{13}\sum_{k=11}^{13}\Gamma_{ik}\langle A_iB_k\rangle\Bigg)+\sum_{k=5}^{10}\langle B_k\rangle+\sum_{i=11}^{13}\langle A_i\rangle-\sum_{i=11}^{13}\sum_{k=5}^{10}\Gamma_{ik}\langle A_iB_k\rangle\\&+\sum_{i=11}^{13}\langle A_iB_i\rangle\leqslant 15,\end{aligned} \tag{3}$$

where $\langle A_iB_k\rangle$ denotes the average in those events where A_i is measured in Alice's side, B_k is measured in Bob's side, as shown in Fig. 1, Γ_{ij} is 1 if $\langle a_i|a_j\rangle=0$, and 0 otherwise. This is a Bell inequality that reveals quantum nonlocality by measuring on two separated system A and B. Any noncontextual hidden variable theories (NCHVTs) satisfies [22]

$$\langle\kappa\rangle\equiv\frac{1}{2}\left(\sum_{i=1}^{4}\langle A_i\rangle-\sum_{i=1}^{4}\sum_{j=5}^{10}\Gamma_{ij}\langle A_iA_j\rangle\right)+\sum_{i=5}^{13}\langle A_i\rangle-\sum_{i=5}^{12}\sum_{j>i}^{13}\Gamma_{ij}\langle A_iA_j\rangle\leqslant 9, \tag{4}$$

where $\langle A_iA_j\rangle$ denotes the average of the product of the outcomes of A_i and A_j in the sequence A_iA_j (A_i first and then A_j). This is a noncontextual (NC) inequality that reveals quantum contextuality by measuring on a local system A.

While the prediction of QM is

$$\langle\beta\rangle_{\rm QM}=15+\frac{2}{3}, \tag{5}$$

which violates inequality (3), and reveals quantum nonlocality between Alice and Bob. And

$$\langle\kappa\rangle_{\rm QM}=9+\frac{2}{3}, \tag{6}$$

which violates inequality (4), and reveals quantum noncontextuality of Alice's local system. To test inequality (3), Alice has to measure $A_i(i=1,\ldots,13)$, and Bob has to measure $B_k(k=4,\ldots,13)$. At the same time, Alice has to measure $A_i(i=1,\ldots,13)$ independently or sequentially on her local system to test inequality (4). These mean that one can observe quantum local contexuality and quantum nonlocality simultaneously. This is different with other theoretical predictions, for example, specified the Bell inequality as CHSH inequality and NC inequality as KCBS inequality [18–20]. The main difference is that the NC inequality (4) is a state independent one, and can be violated using any quantum state, even using maximally mixed state. Thus, when one only measures the local system of Alice, the local state is a maximally mixed state, and can still observe a violation of inequality (4). While for KCBS inequality, it is impossible to observe a violation using maximally mixed state. Considered that the original BKS theory is state independent, inequality (4) is sufficient to reveal quantum contextuality. The simultaneous observation of quantum contextuality and quantum nonlocality connects the two most striking phenomenons of quantum systems.

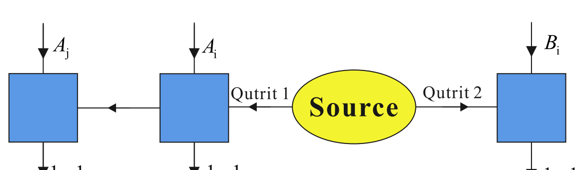

Fig. 1. (Color online) Experimental scheme. Alice performs two compatible dichotomic measurements sequentially, for example, A_i, A_j on her system (qutrit 1), while Bob performs a single measurement, for example, B_k on his system (qutrit 2). According to QM, $\langle\kappa\rangle$ in Alice's side equals to $9+2/3$ for any state and violates a state-independent BKS inequality, $\langle\beta\rangle$ equals to $15+2/3$ when the initial state is a maximally entangled state as shown in Eq. (1) and violates a Bell inequality.

3. Experimental test

To observe quantum contextuality and nonlocality simultaneously, we prepare the maximum qutrit-qutrit entanglement state $|\Psi\rangle=1/\sqrt{3}(|00\rangle+|11\rangle+|22\rangle$ in Eq. (1) with the qutrit encoded in both polarization and spatial mode of photons like in Ref. [23]. The system always consists of one photon with two paths. In one path (the upper path), the photon can be a superposition of horizontal component $|H\rangle$, called state $|1\rangle$, and vertical component $|V\rangle$, called state $|2\rangle$. In the other path (the lower path), the photon is always polarized horizontally. When the photon is in the lower path, it is said to be in state $|0\rangle$. Thus, these states: $|0\rangle, |1\rangle$, and $|2\rangle$ form the qutrit's basis. As illustrated in Fig. 2, our experimental setup consists of three modules: state preparation, Alice's measurement, and Bob's measurement. In the state preparation module, entangled photons with wavelength of 808 nm are generated from type-I spontaneous parametric down-conversion (SPDC) processes, where two joint 0.3 mm-thick β-barium borate (BBO) crystals are pumped by a continuous wave diode laser with 100 mW of power. By adjusting the angels of half wave plates (HWPs), the entangled state can be prepared on $|\varphi\rangle=1/\sqrt{5}|HH\rangle+2/\sqrt{5}|VV\rangle$. After beam displacers (BDs), BD1 and BD2, the entangled state is transformed to $|\varphi_s\rangle=1/\sqrt{5}|H\rangle_3\otimes|H\rangle_4+2/\sqrt{5}|V\rangle_1\otimes|V\rangle_2$. After HWPs in path 1–4 as shown in Fig. 2, the state evolves into $|\varphi_s\rangle=1/\sqrt{5}(|H+V\rangle_1\otimes|H+V\rangle_2+|H\rangle_3\otimes|H\rangle_4)$. After PBS1, the two photon state is post-selected to $|\varphi_s\rangle=1/\sqrt{3}(|H\rangle_5\otimes|H\rangle_6+|V\rangle_5\otimes|V\rangle_6+|H\rangle_7\otimes|H\rangle_8)$ if we only count the coincidence between Alice and Bob. If we encode $|H\rangle$ ($|V\rangle$) in path 5 and 6 into $|1\rangle$ ($|2\rangle$), and encode $|H\rangle$ in path 7 and 8 into $|0\rangle$, the qutrit-qutrit entangled state $|\varphi_s\rangle=1/\sqrt{3}(|00\rangle|+|11\rangle+|22\rangle)$ is prepared.

To measure Alice's observation A_i, A_j and their correlations, we use cascaded Mach-Zehnder interferometers in three steps [23]. The first step is to realize the measurement of A_i with four HWPs (HWP1–4) and BD3. The angles of HWP1 and HWP2 are chosen properly so that the photons which are horizontally (vertically) polarized after BD4 corresponding to eigenvalue -1 (1).

Measuring A_iA_j requires two sequential measurements on the same photon. Since the single-observable measuring devices map its eigenstates to a fixed spatial path and polarization, with HWPs and BDs, we can recreate the corresponding eigenstates of A_i for further measurement A_j in the second step.

In the third step, we use the same devices as in the first step to measure A_i. Two identical A_i measurement devices (one is constructed by HWP9–10, BD6, and another by HWP13–15, BD8) are built, each of them is connected to the corresponding output port

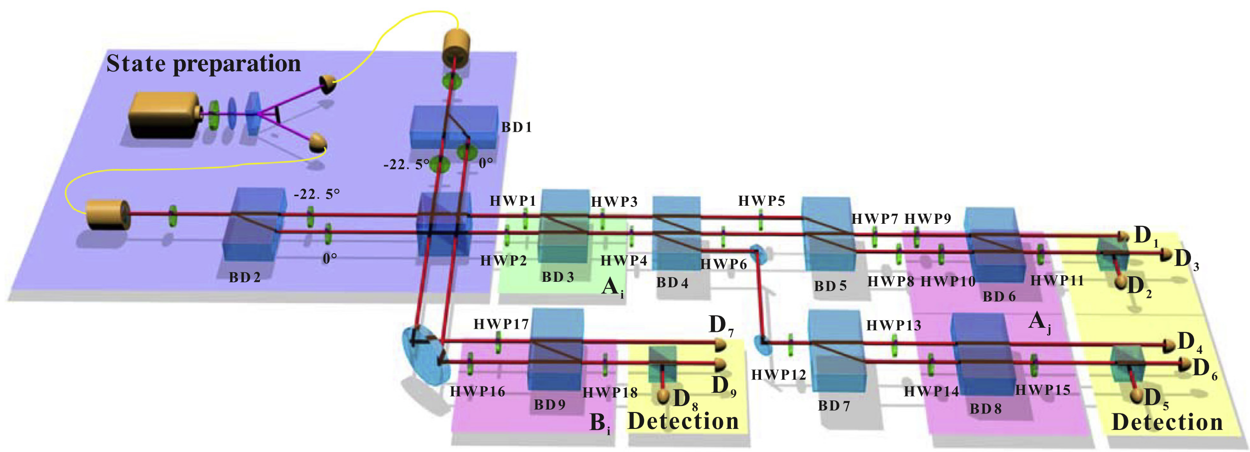

Fig. 2. (Color online) Experimental setup. Entangled photons $1/\sqrt{2}(|HH\rangle + |VV\rangle)$ are generated via type-I spontaneous parametric down-conversion process. Four half wave plates set at 0° or 22.5° and BD1, BD2 are used to prepare post-selected qutrit-qutrit entangled state $|\Psi\rangle = (|00\rangle + |11\rangle + |22\rangle)/\sqrt{3}$. On Alice's side, cascaded setup for sequentially measuring A_i and A_j is used to test the noncontextual inequality (4) and detection on Bob's side is set as a trigger. To test the Bell inequality (3), Alice performs the same measurement on her side and Bob performs B_k on his side. All the measurement results can be calculated by the two folds coincidence between Alice's side and Bob's side. BBO-β-barium borate, HWP-half wave plate, BD-beam displacer, PBS-polarizing beam splitter.

of the measurement device of A_i. The outcomes of the measurement $\langle A_iA_j\rangle$ are given by the responses of detectors $D_1 - D_6$.

For Bob, the measurement of observable B_k is the same as A_i using HWPs and BDs. As we have mentioned before, HWP16–18 is used to map the photon state to horizontally (vertically) polarizing state which is corresponding to the eigenvalue $-1\,(1)$. The photons are detected by detectors $D_7 - D_9$.

We only register the coincidence rates between the detectors of Alice and Bob with a coincidental window of 3.2 ns. For each measurement, we record clicks for 60 s, and the total coincidence is approximate 30,000. To test NC inequality, the correlation $\langle A_iA_j\rangle$ is constructed from four measured joint probabilities $P(A_i = \pm1, A_j = \pm1)$. Similarly, we can evaluate the value of β_{AB} for the Bell inequality with the correlation $\langle A_iB_k\rangle$ which is constructed from the measured joint probability $P(A_i = \pm1, B_k = \pm1)$. All the joint probabilities can be read out from the coincidence between certain detectors of Alice and Bob. All of the HWPs setting and detector coincidence are listed in the Supplementary data.

The experimental results on the mean values of β and κ are

$$\begin{aligned}\langle\kappa\rangle_{\rm QM} &= 9.560 \pm 0.026,\\ \langle\beta\rangle_{\rm QM} &= 15.461 \pm 0.022.\end{aligned} \tag{7}$$

Our experimental results show that $\langle\kappa\rangle_{\rm QM} = 9.560 \pm 0.026$ which is closed to the quantum mechanics's prediction of 9 + 2/3, and exceeds the NCHVT's bound of 9 by 22 standard deviations. At the same time, $\langle\beta\rangle_{\rm QM} = 15.461 \pm 0.022$ which is closed to the quantum mechanics's prediction of 15 + 2/3, and exceeds LHVT's bound of 15 by 21 standard deviations. Thus we observe quantum nonlocality and quantum contextuality simultaneously. All the experimental details are listed in the Supplementary data. The overall detection efficiency is approximate 4.6% and we have to employ the assumption of fair sampling [24].

4. Discussion and conclusion

The relation between quantum contextuality and quantum nonlocality was a fundamental and confusing problem in QM. There are three relations between quantum contextuality and quantum nonlocality: nonlocality-nonlocality, contextuality-contextuality and nonlocality-contextuality. Nonlocality-nonlocality relation is first studied and obey the interesting property of Bell monogamy [25,26]. This property is useful in secure quantum key distribution [27], interactive proof systems [28], and in the emergence of a local realistic description for correlations in the macroscopic domain [29,30]. However, someone noticed that monogamy violations become generic with increasing dimension of the system [31,32]. Contextuality-contextuality has similar relationship with nonlocality-nonlocality, studied by Ref. [33] using graph-theoretic technique from the no-disturbance principle. Recently, it has been recognized that both quantum nonlocality and quantum contexuality can be used as quantum resources for quantum communication and quantum computation. This observation puts the problem of what is the relation between contextuality and nonlocality under a new perspective: quantum information technique. It raises the question of whether single-particle contextuality and two-particle nonlocality can coexist, such that the same quantum system can provide both resources simultaneously. The answer is "NO" if we restrict ourselves to simple forms of nonlocality and single-particle contextuality and it have a monogamy between them [18–20]. Here, we give a counter example if we use state-independent NC inequality. However, how to utilize these two quantum resources simultaneously is still an open question.

In conclusion, we find an example that breaks the monogamy relation between quantum contextuality and quantum nonlocality, and observe violations of Bell inequality and NC inequality simultaneously in a photonic qutrit-qutrit system. This give a new perspective to investigate the nonlocality-contextuality relation. Our experiment methods can be easily scaled to higher dimension and opens the door to experimentally observe the relationship between quantum nonlocality and quantum contextuality.

Conflict of interest

The authors declare that they have no conflict of interest.

Acknowledgments

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.scib.2018.06.018.

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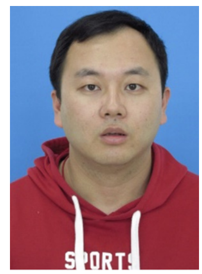

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