

MICHELL, LAPLACE AND THE ORIGIN OF THE BLACK HOLE CONCEPT

Colin Montgomery, Wayne Orchiston and Ian Whittingham

Centre for Astronomy, James Cook University, Townsville,
Queensland 4811, Australia

E-Mail: Colin.Montgomery@jcu.edu.au;

Wayne.Orchiston@jcu.edu.au;

Ian.Whittingham@jcu.edu.au

Abstract: Black holes are fundamental to our understanding of modern astrophysics, yet the origin of this concept can be traced back to the writings of England's John Michell and France's Pierre-Simon Laplace in 1784 and 1796 respectively. Both independently postulated the existence of "non-luminous bodies", and while Michell used graphical methods to explain his concept, Laplace published a mathematical 'proof' in 1799.

Key Words: Michell, Laplace, black holes, missing mass, dark matter

1 INTRODUCTION

The concept of a black hole is now widely accepted in astronomy and is applied to a range of bodies, extending from extremely small primordial black holes formed during the Big Bang with masses less than the mass of the Earth, to stellar-remnant black holes with masses of 3-30 M_{\odot} resulting from supernova explosions, up to supermassive black holes with masses of 10^6 - $10^9 M_{\odot}$ at the centres of active galaxies. Such objects, defined by the common characteristic that their gravitational fields are so strong that light cannot escape from them, comprise a part of the baryonic component of dark matter in the Universe and are regarded as having their theoretical foundation in Einstein's General Theory of Relativity.

However, the concept of black holes has a much longer history, dating back to studies by Britain's Reverend John Michell and France's Pierre-Simon Laplace in the eighteenth century (see Hodges, 2007; Israel, 1987: 110, 201-203; Schaffer, 1979). In this paper we discuss their work, and also provide a modern version of Laplace's mathematical 'proof' of the existence of 'invisible bodies' such as black holes. It is important to recall that at that time light was believed to consist of corpuscles (rather than wave motion).

2 JOHN MICHELL

2.1 A Biographical Sketch

The Reverend John Michell was the son of the Rector at Eakring in central Nottinghamshire, and was born at the Rectory on Christmas Day in 1724 (Crossley, 2003). He was admitted to Queens' College, Cambridge, in 1742, graduating in mathematics as Fourth Wrangler in 1748. The following year he was elected a Fellow of Queens' College, where he taught arithmetic, geometry, Greek and Hebrew. He obtained an M.A. in 1752 and a B.D. in 1761, and was elected a Fellow of the Royal Society in 1760. In 1762 he was appointed Woodwardian Professor of Geology at Cambridge, but just one year later he became Rector of Compton, near Winchester, and spent the rest of his life as a clergyman. Later he moved to Yorkshire, where he carried out the astronomical studies that are the focus of this paper. John Michell died on 29 April 1793 (Hoskin, 2004).¹

There is no known image of John Michell but we have a description of him which was recorded in Cole MSS XXXIII, 156, in the British Library:

John Michell, BD is a little short Man, of a black Complexion, and fat; but having no Acquaintance with him, can say little of him. I think he had the care of St. Botolph's Church, while he continued Fellow of Queens' College [Cambridge], where he was esteemed a very ingenious Man, and an excellent Philosopher. (Cited in Crossley, 2003).

Although known as the 'father of modern seismology', Michell was a polymath and made important contributions in a number of fields of science (see Hardin, 1966), including geology and astronomy. Hughes and Cartwright (2007: 93; their italics) claim that Michell was "... the first *statistical* astronomer, and that he pioneered the application of probability theory to stellar distributions."

Jungnickel and McCormach (1996: 301) also sing Michell's praises:

... his publications in astronomy were—by default, it would seem—theoretical. In speculative verve he was Herschel's equal, and since he had mathematical skills equal to Maskelyne's and Cavendish's, he could develop his theoretical ideas farther. In breadth of scientific knowledge, Michell resembled William Watson ... like Watson, Michell was knowledgeable in natural history as well as in natural philosophy.

As an astronomer, Michell was both an observer and a theoretician. During his life he made at least one telescope, a large reflector, in about 1780. Soon after Michell's death this instrument was described by his son-in-law in a letter to William Herschel:

The dimensions & state of the telescope are nearly as follows. A Reflecting Telescope Tube 12ft long made of Rolled Iron painted inside and out, & in good preservation. The Diameter of the large Speculum 29 inches. Focal length 10 feet, its weight is 330 lbs it is now cracked. There are also 8 concave small mirrors of different sizes ... and 2 convex mirrors ... there are also [?] sets of eyeglasses in brass tubes & cells. The weight of the whole is about half a tun [*sic*] ... (Turton, 1793).

Herschel went on to purchase this telescope, but this was his only association with Michell. Hutton (2006) has shown that there is no validity to the claim advanced by one of Michell's descendants in 1871 that it was John Michell who inspired Herschel to take up astronomy.

2.2 Michell's First Astronomical Publication

Michell's first paper was published by the Royal Society in 1767, and was concerned with the distances of stars based upon their parallaxes, and with the true nature of double stars. This paper was in response to Pierre Bouguer's *Traité d'Optique sur la Gradation de la Lumière* which placed importance on the distinction between the quantity and intensity of light. Michell applied a new approach to British sidereal astronomy, believing that the mass of a star determined the quantity of light it emitted. He based his argument upon a pioneering probability analysis, and demonstrated that nearly all double stars were binary systems and not chance alignments (i.e. optical doubles).

Michell proposed a purely dynamic procedure for determining the mass and density of binary stars, as summarised by McCormmach (1968: 139):

According to gravitational theory, the period and greatest separation of a double star determine the relation between the apparent diameter and density of the central star. If the distance of the central star is somehow known, its apparent diameter can be converted into its true diameter. The mass and the surface can be calculated from the true diameter and density, and the star's total light and brightness are then referred to these magnitudes.

Hughes and Cartwright (2007) provide a succinct statistical examination of Michell's paper, which Hoskin (2004) has described as "... arguably the most innovative and perceptive contribution to stellar astronomy to be published in the eighteenth century."

When he began a search for new double stars in 1779, Herschel apparently was unaware of Michell's seminal paper of 1767, and in 1782 he published his "Catalogue of double stars" in the *Philosophical Transactions of the Royal Society* (Herschel, 1782a), along with a paper about stellar parallaxes (Herschel, 1782b).

2.3 Michell Introduces the Concept of Black Holes

Herschel's two 1782 papers, and especially the catalogue of double stars, provided Michell with the 'means' on which to base his second astronomical paper, which bears the exceedingly long and laborious title, "On the Means of Discovering the Distance, Magnitude, &c. of the Fixed Stars, in Consequence of the Diminution of the Velocity of Their Light, in Case Such a Diminution Should be Found to Take Place in any of Them, and Such Other Data Should be Procured from Observations, as Would be Further Necessary for That Purpose."

Michell completed this paper in May 1783 and sent it to his London-based friend, Henry Cavendish (Figure 1),² who at the time was regarded as the Royal Society's "... scientifically most eminent member." (Jungnickel and McCormmach, 1996: 249; their italics). Cavendish showed the paper to Maskelyne, Herschel, and other members of the Royal Society, and he read the paper—in three instalments—at the 11 and 18 December 1783 and 15 January 1784 meetings of the Society. This was a time when the Society was in turmoil as two opposing groups of members fought respectively to retain and unseat the President, Sir Joseph Banks, and Michell's paper was the only one read at the two December meetings (*ibid.*: 249-256). Apparently, Michell was in the habit of regularly

making the long journey from Yorkshire to London in order to attend meetings (*ibid.*: 301), so it is strange that he decided not to present this important paper himself. Maybe the disruptive nature of the Society's meetings at this time prompted him to stay away from London. Alternatively, his paper was speculative, so perhaps he felt that it would gain greater acceptance by his peers if presented by his illustrious colleague.



Figure 1: Sketch of Henry Cavendish, 1731–1810 (after http://en.wikipedia.org/wiki/File:Cavendish_Henry.jpg).

Be that as it may, Michell opens his discussion with the observation that Herschel had discovered a large number of double and triple stars. Referring to his own 1767 paper, Michell suggests that these stars would be affected by their mutual gravitational attraction, and that observations should reveal the period of revolution of the secondary components in some of these systems. In a binary system, if the diameter of the central component, the separation of the two components and the period of revolution of the secondary component are all known, then the density of the central component can be calculated. Knowing the density of any central body and the velocity any other body would acquire by falling towards it from an infinite height, then the mass and size of the central body can be calculated. Michell then suggests that particles (corpuscles) of light are attracted by gravitational forces (just like celestial bodies), so if any star of known density is large enough to affect the velocity of light issuing from it, then we have a means of calculating its actual size.

Michell then proceeds to carry out a geometrical analysis of the various velocities and forces that would apply. Referring to Figure 2, Newton's 39th proposition in *Principia* can be illustrated with respect to the velocity the body acquires falling towards the central body, C, by constructing perpendiculars, such as rd, to the directional line in proportion to the force applied at that point. The velocity acquired at that point is then proportional to the square root of the area described, for example AdrB.

If C is the centre of the central body attracting the falling body from infinity, A , then RD represents the force applied to the falling body at point D and the velocity acquired at D is the same as that acquired in falling from D to C under the force RD , where RD is inversely proportional to the square of DC , provided the area of the infinitely-extended hyperbolic space $ADRB$ is equal to rectangle RDC .

The velocity of a falling body at the same distance from C will be proportional to the square root of the density of the central body, as the distance Cd will remain constant and rd will change in proportion to the density, and the rectangle rdC proportionally.

As the masses of different spheres of the same density are determined by their radii, the rectangles RDC and rdC will be increased or reduced in the square ratio of the radius CD and consequently the velocity in the simple ratio of CD .

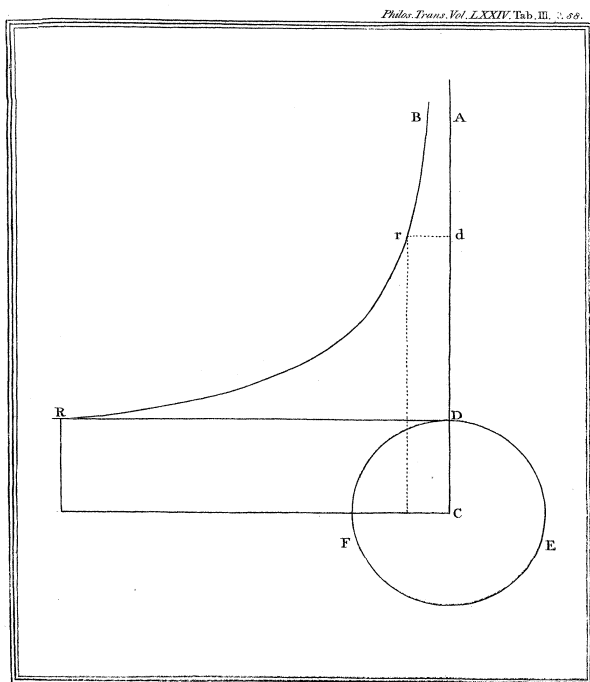


Figure 2: The explanatory diagram that accompanied Michell's 1784 paper.³

If the velocity of a falling body is the same when it reaches the surface of two different central bodies, then the rectangle RDC will remain the same. RD must be inversely proportional to CD , and the density is therefore inversely proportional to CD^2 .

In modern parlance, Michell's diagram (Figure 2) is a graph of the gravitational force, $F = GMm/r^2$ between the falling body of mass, m , and the central attracting body of mass, M , as a function of the distance, r , from the central body. The area is the work done by the gravitational force, and is equal to the gain in kinetic energy of the mass, m . For an initially-stationary mass, m , free-falling from an infinite height, the work done (area $ADrB$) is GMm/r and the acquired velocity is given by $v^2 = 2GM/r$. At the surface of M , the area $ADRB$ is GMm/R , the same as the rectangle RDC . If the central body is a sphere of density ρ , then $F = k\rho mR^3/r^2$, where $k = 4\pi G/3$, and the velocity of the falling body is $v^2 = 2k\rho R^3/r$, that is, v^2 is proportional to ρ at a fixed distance r . At the surface

of M this becomes $v^2 = 2k\rho R^2$, and will be the same at the surfaces of different central bodies if ρR^2 is constant, that is, the density must be inversely proportional to CD^2 .

Michell knew that the velocity of the falling body at the surface of the Sun is the same as a comet revolving in a parabolic orbit at the Sun's surface, but with a velocity 20.72 times the velocity of the Earth in its orbit of 214.64 times the Sun's radius. As the square of the velocity of the comet is twice the square of the velocity of a planet, the ratio of the squares of the velocities is 429.28:1, and the square root of 429.28 is 20.72.

Michell considered the speed of light to be 10,310 times the Earth's orbital velocity, which if divided by 20.72 gives approximately 497. This is the number of times the velocity of light would exceed the velocity of a body falling from infinity onto the surface of the Sun, and the ratio of an area whose square root should exceed the square root of area RDC , RD being the force of gravity at the surface of the Sun and CD the Sun's radius.

Therefore

... if the semi-diameter of a sphere of the same density with the sun were to exceed that of the sun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its surface a greater velocity than that of light, and consequently, supposing light to be attracted by the same force in proportion to its vis inertiae, with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity. (Michell, 1784: 42)

In this scenario, the central body would remain invisible, and using modern parlance we would refer to it as a black hole.

Michell then proceeds to comment that if the diameter of a sphere was <497 times that of the Sun, light would escape, but at a very much reduced velocity.

He notes that it is difficult to determine the distance to individual stars and groups of stars that are at very large distances, except when these groups include double and triple stars to which his analysis can be applied. He suggests that it will be many years—even decades—before new double and triple stars will be found in sufficient numbers to test his theory. Since the revolution of some secondary components about their central stars takes many years, Michell expresses the hope that relevant observations of double and multiple stars will be made by future generations.

Michell (1784: 50) further suggests that

If there should really exist in nature any bodies, whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us; or if there should exist any other bodies of a somewhat smaller size, which are not naturally luminous; of the existence of bodies under either of these circumstances, we could have no information from light; yet, if any other luminous bodies should happen to revolve about them we might still perhaps from the motions of these revolving bodies infer the existence of the central ones with some degree of probability, as this might afford a

clue to some of the apparent irregularities of the revolving bodies, which would not be easily explicable on any other hypothesis ...

It is interesting that this method is now widely used by contemporary astronomers to search for black holes. The X-ray sources Cygnus X-1, LMC X-3 and V404 Cygni were identified as stellar remnant black holes through observations of their optical binary companions, and the evidence for a $4 \times 10^6 M_{\odot}$ supermassive black hole at the centre of our galaxy comes from the analysis of short period stellar orbits about SgrA* (see Reid, 2009).

3 PIERRE-SIMON LAPLACE

3.1 A Biographical Sketch

Pierre-Simon Laplace (Figure 3) is considered one of France's greatest scientists (see Gillispie, 1997; Hahn, 2005). He was born on 23 March 1749 in lower Normandy where his father was a syndic of the parish. He began his education at the college at Beaumont-en-Auge, where his uncle taught, remaining there until he reached the age of sixteen. From this college students normally proceeded into the army or into an ecclesiastical vocation, and in 1766 Laplace moved to the University of Caen, where he matriculated in the Faculty of Arts after just two years.

During this two-year period Laplace discovered that he possessed mathematical gifts, so he abandoned his theological studies and in 1768 moved to Paris. There he came under the watchful eye of d'Alembert, a leading scientist in the French Academy, who obtained for him the appointment of Professor of Mathematics at the École Militaire. Laplace taught there from 1769 to 1776, and during this interval he presented thirteen papers on mathematics and the theory of probability to win election to the Academy of Science. One of the papers was on "The Newtonian theory of the motion of planets", which Laplace translated into Latin. He was elected to the Academy in 1773.

This was the era of the French Revolution (1789-1799), a period of political and social upheaval throughout France as it moved from an absolute monarchy with feudal privileges for the aristocracy and the Catholic clergy to a form based on the enlightenment principles of nationalism, citizenship and inalienable rights.

Laplace had remained at the Academy, and the fall of Robespierre and the Jacobin regime in 1794 saw a dramatic change in the education system which led to the institutionalization of modern French society. Various institutions of science emerged (including the Institute de France), but some of these fell by the wayside only to re-emerge at a later stage. In 1795 Laplace was elected Vice-President of the Institute of France, and a year later was made President.

In this position he deferred from giving lectures at the Institute, and instead referred the auditors to a book he was preparing titled *Exposition du Système du Monde* which appeared in two volumes in 1796.

Pierre-Simon Laplace died on 5 March 1827, just two and a half weeks short of his 78th birthday.

3.2 Laplace Independently Introduces the Concept of Black Holes

In the sixth chapter of *Exposition du Système du Monde* Laplace introduced speculation as to the origin of the Solar System and the nature of the Universe.

The Sun lies in the centre of the Solar System and spins on its axis every twenty-five and a half days and its surface is covered with 'oceans' of luminous matter spotted with dark patches. The atmosphere above this extends beyond recognition. Around the Sun spin the seven planets in almost circular orbits. However, Laplace did not consider comets to be part of the Solar System as some travelled in highly eccentric orbits, and while they moved into the Sun's domain they also moved far beyond the planetary sphere.



Figure 3: Pierre-Simon Laplace, 1749–1827 (after http://en.wikipedia.org/wiki/Pierre-Simon_Laplace#References).

In describing the Solar System Laplace (1796: 305) makes another conjecture:

The gravitation attraction of a star with a diameter 250 times that of the Sun and comparable in density to the earth would be so great no light could escape from its surface. The largest bodies in the universe may thus be invisible by reason of their magnitude.⁴

Laplace stated this possibility in a merely qualitative way—almost in passing—without any mathematical proof, and he only proceeded to provide the latter when asked to do so by F.X. von Zach. This was subsequently published in the German journal, *Allgemeine Geographische Ephemeriden* (Laplace, 1799),⁵ which von Zach edited.

3.3 Laplace's Mathematical 'Proof'

Laplace's (1799) proof of the existence of 'invisible bodies' (or black holes) took the form of an essay, which we summarise below.

For non-uniform motion, the velocity v over the time interval dt must be taken as

$$v = \frac{dr}{dt} \quad (1)$$

where dr is the distance travelled.

A continuously working force will strive to change the velocity. This change in velocity, namely dv , is therefore the most natural measure of the force. But as any force will produce double the effect in double the time, so we must divide the change in velocity dv by the time dt in which it is brought about by the force P (see Note 6), namely

$$P = \frac{dv}{dt} = \frac{d}{dt} \left(\frac{dr}{dt} \right) = \frac{d^2r}{dt^2} \quad (2)$$

The attractive force between a body M and a particle of light at a distance r is proportional to $-M/r^2$ (also see Note 6). The negative sign occurs because the action of M is opposite to the motion of the light. Equating P with $-M/r^2$ and integrating gives

$$v^2 = 2C + 2Mr^{-1} \quad (3)$$

where v is the velocity of the light particle at the distance r .

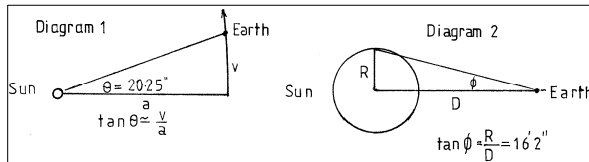


Figure 4: Diagrams used to help explain Laplace's concept of 'black holes'.

To determine the constant C , let R be the radius of the attracting body, and a the velocity of the light at the distance R . Hence on the surface of the attracting body one obtains

$$2C = a^2 - 2M/R \quad (4)$$

so that

$$v^2 = a^2 - \frac{2M}{R} + \frac{2M}{r} \quad (5)$$

Let R' be the radius of another attracting body with attractive power iM . The velocity of light at a distance r will be v' ,

$$v'^2 = a^2 - \frac{2iM}{R} + \frac{2iM}{r} \quad (6)$$

As the distance of the fixed stars is so large, one can make r infinitely large, and one obtains

$$v'^2 = a^2 - \frac{2iM}{R'} \quad (7)$$

Let the attractive power of the second body be so large that light cannot escape from it; this can be expressed analytically as the velocity $v' = 0$ at an infinitely large distance. This gives

$$a^2 = \frac{2iM}{R'} \quad (8)$$

To determine a , let the first attracting body be the Sun; then a is the velocity of the Sun's light on the surface of the Sun. The gravitational force at the surface of the Sun is so small that its effect on the velocity of light leaving that surface can be neglected in the context of this discussion.

Laplace (1796) uses his assumption made on page 305 in *Exposition du Système du Monde (Part II)*, that $R' = 250R$. Since the mass changes as the volume of the attracting body multiplied by its density and therefore as the cube of the radius, then, if the density of the Sun is 1 and that of the second body is ρ ,

$$M:iM = 1R^3 : \rho R'^3 = 1R^3 : \rho 250^3 R^3 \quad (9)$$

or

$$i = (250)^3 \rho \quad (10)$$

Substitution of the values of i and R' in the equation (8) gives

$$\rho = \frac{a^2 R}{2(250)^2 M} \quad (11)$$

for the density of the body from which light cannot escape. To obtain ρ , one must still determine M . The force of the Sun is equal at a distance D to M/D^2 . If D is the average distance of the Earth and V the average velocity of the Earth, then this force is also equal to V^2/D (see Lalande 1792, Volume 3: 3539). Hence

$$M/D^2 = V^2/D \quad (12)$$

or

$$M = V^2 D \quad (13)$$

Substituting this into equation (11) gives

$$\rho = \frac{8}{(1000)^2} \left(\frac{a}{V} \right)^2 \left(\frac{R}{D} \right) \quad (14)$$

From the phenomena of aberration, it appears that the Earth travels $20.25''$ in its path while light travels from the Sun to the Earth. Referring to Figure 4, the ratio a/V , the velocity of light divided by the velocity of the Earth, is given by

$$a/V = 1/\tan 20.25'' \quad (15)$$

R/D is the absolute radius of the Sun divided by the average distance of the Sun, and is equal to the tangent of the average apparent angular radius of the Sun, which is $\tan 16' 2''$ (see Figure 4).

Hence the required density is given by

$$\rho = 8 \tan 16' 2'' / (1000 \tan 20.25'')^2 \quad (16)$$

which is approximately 4, or about that of the Earth.

4 DISCUSSION

It is important to note that Michell's reference to what we would now call a 'black hole' was merely a by-product of his 1784 paper and not the main focus of that paper. The paragraph containing the description of a black hole developed out of his theory about the distance to and relative sizes of double stars, although it is worth pointing out that on page 50 Michell (1784) also describes the perturbations of stars by "... bodies of a somewhat smaller size, which are not naturally luminous ..."

As we have seen, just twelve years later the Frenchman, Laplace, followed up Michell's work by independently proposing the existence of 'black holes' and three years on he provided the mathematical proof of these. However this would appear to be a remarkable coincidence as there was little scientific contact between England and France during this extremely troubled time in French history. Thorne (1994), amongst others, has suggested otherwise: that upon

hearing of Michell's theory that gravity could prevent light from emerging from a star, Laplace immediately proceeded to provide the mathematical proof. Furthermore, Thorne (*ibid.*) has stated that the reason that Laplace did not include the proof in the original edition of his book, *Exposition du Système du Monde*, and excluded it from several subsequent editions was that he did not believe in the existence of 'black holes'.

The first of these claims is not substantiated by Laplace's biographer, Charles Coulston Gillispie (1997) and Laplace's 1799 paper. The latter paper leaves no doubt that it was von Zach who requested that Laplace provide a mathematical proof to the simple statements made in his 1796 book.

Our British colleague, Emeritus Professor David W. Hughes (pers. comm., 2009), has made the following pertinent comment:

It is interesting that Michell and Laplace both 'backed the wrong horse' when it came to predicting what stellar black holes might be like. Looking at the formula for the escape velocity one can see that one can have a black hole if you have a star of solar density that is very large. Or you can have a black hole if you have a star that is a bit more massive than the Sun but has a very small size and thus a very high density.

Both Michell and Laplace went for the 'big star' option. But this was wrong. The black holes that have been found are all very small size and very high density.

Why did they get it wrong? Maybe it was just down to the physics of the day. The most dense material at the time was gold and having substances much more dense than this was probably thought to be impossible. When it came to stellar size they had only one point on their graph, the diameter of the Sun. No other stellar sizes were known. They thought that there might be stars bigger than the Sun but seemed reluctant to consider the possibility that there might be much smaller ones. Let's face it, the physics of white dwarfs must have been surprising in the late 19th century, just as the physics of black holes is today.

Finally, it is of interest to reflect on the similarity in the lives of Michell and Laplace. While both studied theology with a view to entering the church, their interests turned towards mathematics and particularly the laws of probability. Indeed, it was the probability that non-luminous bodies should exist that led the two scientists quite independently to the concept of a 'black hole'. Having said that, it is important to realise that the non-luminous bodies postulated by Laplace were much larger than those suggested by Michell. The Laplacian body had a radius of $250 R_{\odot}$, a density four times that of the Sun, and consequently a mass of $4 \times (250)^3 M_{\odot}$, about 10^7 times the $500 M_{\odot}$ body considered by Michell.

5 CONCLUDING REMARKS

The talented English scientist turned clergyman, John Michell, was the first to postulate the existence of a black hole in 1784 when he published a paper in the *Transactions of the Royal Society* which dealt with the distances of double stars and the relative sizes of their components. By a strange twist of fate, France's Pierre-Simon Laplace independently postulated the existence of black holes in his book, *Exposition du Système du Monde*, which was published in 1796,

and three years later he published the necessary mathematical proof—not in response to Michell's earlier paper, but because the noted German astronomer F.X. von Zach specifically requested it.

6 NOTES

1. Note that Hodges (2007) gives Michell's date of death as 21 April rather than 29 April, but we have opted for the latter date.
2. According to Jungnickel and McCormmach (1996: 139),

Michell and Cavendish's acquaintanceship, if not their friendship, began no later than ... 1760. That year, at Cavendish's first dinner as a member of the Royal Society Club, Michell was present as a guest, and in later years Cavendish often brought Michell as his own guest. In 1760, Michell and Cavendish were both elected Fellows of the Royal Society ...

Michell and Cavendish often took the opportunity to discuss their different philosophies on the appearance of the inverse square law in nature (Hardin, 1966). As early as 1750 Michell had stated the mathematical properties of magnetic force and in 1771 Cavendish established the laws of electrical attraction and repulsion, both related to the inverse square law (Jungnickel and McCormmach, 1996).

3. This figure appeared in the published version of Michell's 1784 paper, but is missing from the MS of the paper in The Royal Society's Archives. We are pleased to report that as a result of a dedicated search on behalf of the first author, Joanna Corden, the Archivist and Records Manager at The Royal Society, recently found the missing figure bound by mistake between the first and second pages of a meteorological paper by John Atkins which immediately follows Michell's paper in the *Philosophical Transactions*. Subsequently, a further copy of this diagram was found at the Cambridge University Library in the William Herschel Papers.
4. It should be understood that the word 'magnitude' is used here in its eighteenth century sense to refer to that which "... can be compared by the same common feature ..." (Bailey, 1737), and therefore differs significantly from our current usage of the term.
5. An English translation of Laplace's 1799 paper appears in Hawking and Ellis (1973: 365-368) as Appendix A.
6. Note that P and M , as used here, are respectively the force per unit mass and the mass of the body multiplied by the Newtonian constant, G .

7 ACKNOWLEDGEMENTS

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Colin Montgomery is an engineer with an passion for cosmology. He has a Master of Astronomy from the University of Western Sydney, and is about to complete a Ph.D. in the Centre for Astronomy at James Cook University with a thesis on "From Michell to MACHOS: Changing Perspectives on our Understanding of Dark Matter" Montgomery, 2009).

Dr Wayne Orchiston is a Reader in the Centre for Astronomy at James Cook University, and a former Secretary of IAU Commission 41 (History of Astronomy). His research interests include the history of radio astronomy, transits of Venus, comets, and historically-significant telescopes and observatories. He has published extensively, and his latest book, on early Australian radio astronomy (co-authored by Woody Sullivan and Jessica Chapman) is scheduled to appear shortly under Springer's banner.

Ian Whittingham is an Adjunct Professor of Physics in the School of Engineering and Physical Sciences at James Cook University, and his research interests include particle-physics motivated models of dark matter and dark energy, spectral lines in brown dwarf atmospheres and ultra-cold collisions of metastable gases.