Observations of acoustic Wannier configurations revealing topological corner anomaly

Peng Zhang\textsuperscript{a,b}, Han Jia\textsuperscript{b,c,*}, Jiuyang Lu\textsuperscript{d,e,*}, Xinghang Yang\textsuperscript{a,b}, Suhao Wang\textsuperscript{a}, Yuzhen Yang\textsuperscript{a}, Zhengyou Liu\textsuperscript{a,d,e,*}, Jun Yang\textsuperscript{a,b,*}

\textsuperscript{a} Key Laboratory of Noise and Vibration Research, Institute of Acoustics, Chinese Academy of Sciences, Beijing 100190, China
\textsuperscript{b} University of Chinese Academy of Sciences, Beijing 100049, China
\textsuperscript{c} State Key Laboratory of Acoustics, Institute of Acoustics, Chinese Academy of Sciences, Beijing 100190, China
\textsuperscript{d} School of Physics and Optoelectronics, South China University of Technology, Guangzhou 510641, China
\textsuperscript{e} Key Laboratory of Artificial Micro- and Nano-Structures of Ministry of Education and School of Physics and Technology, Wuhan University, Wuhan 430072, China

A R T I C L E   I N F O

Article history:
Received 22 November 2022
Received in revised form 13 February 2023
Accepted 7 March 2023
Available online 11 March 2023

Wannier configuration, which is the real-space distribution of the spatial locations of symmetric Wannier functions (SWFs), provides a new viewpoint for the wave motions in crystalline structures. In Wannier configurations, the deviation of the spatial location of SWF from the unit-cell center often indicates the existence of nontrivial boundary signatures. Moreover, the excess charges extracted from Wannier configuration constitute genuine topological indicators in real space [1,2]. In two-dimensional higher-order topological insulators (HOTIs) [3–5], the scheme of Wannier configurations can manifest fractional charges to reveal the corner modes even when these modes emerge in the bulk bands. Hence, Wannier configurations are of particular importance in classical metamaterials due to the lack of the chiral or particle-hole symmetry which is essential to pin the corner modes into the mid-gap [6]. In optical topological systems, Wannier configurations have been utilized to detect the topological crystalline classification [7], experimentally verify the emergence of half quantum number [8], and characterize hierarchical bulk topology in a system without complete bandgap [9].

In phonic crystals (PCs) where identifying acoustic HOTIs highly relies on searching the corner modes lying within the bandgap [10–15], a pioneering work has proposed the observation of zero-energy states residing in the bulk continuum by measuring local density of states (LDOS) [16]. However, such zero-energy states highly depend on the generalized chiral symmetry achieved by adding fine-tuned auxiliary resonators to the boundary as a compensation. For many crystals with complex structures, such boundary compensation requires precise sample design and fabrication. In all, it is still a challenging problem to capture higher-order topology when corner modes are merged into the bulk bands.

In this letter, we report an observation of acoustic Wannier configurations. Two topologically distinct Wannier configurations, namely Phase A (Fig. 1a) and Phase B (Fig. 1d), are constructed in the two-dimensional (2D) PCs with fourfold rotation symmetry. For each phase, we integrate LDOS spectra over the spectrum band to obtain the spectral charges. The resultant Wannier configurations of Phases A and B manifest fractional and zero corner charge, respectively. The measured fractional spectral charges can characterize topological corner anomaly, revealing the corner mode regardless of its frequency. It is worth noting that the proposed PCs of Phases A and B do not require fine-tuned boundary compensations and are robust to the possible experimental deformations. Furthermore, the Wannier configurations in different phases are naturally combined to reproduce the corner modes in band gaps. Both the ordinary and the exotic phases can serve as the cladding layers in the combined PCs, providing an anomalous route to engineering in-gap corner modes.

We first realize acoustic Wannier configurations in the PC in Phase A with a 6 × 6 array of unit-cells (Fig. 1a), where the lower-left corner unit-cell is magnified in the inset to guide eyes. Four acoustic cavities are coupled by the thick tubes to form one bulk plaquette (brown). The bulk plaquettes are terminated by two intersected boundaries at the corners, leading to the fractionalized corner plaquettes (red). On the other hand, the edge plaquettes (blue) are the results of the terminated bulk plaquettes by the

\[ \text{https://doi.org/10.1016/j.scib.2023.03.015} \]

2095-9273/© 2023 Science China Press. Published by Elsevier B.V. and Science China Press. All rights reserved.
boundary along $x$ or $y$ direction. Acoustic fields in these plaquettes, which are the linear combinations of the nodeless modes in the cavities, manifest as $s, p_x/p_y$ and $d$ orbitals, i.e., three types of SWFs admitted in group $p4$. These SWFs correspond to respective acoustic dispersions hosting specific symmetry labels in momentum space (the Supplementary materials Note 1). Therefore, we can use the locations and charges of SWFs to illustrate specific Wannier configuration.

In classical metamaterial systems, the charge of SWFs can be characterized by the integrated LDOS over a spectral band. Equivalent to the charge given by the band filling in electronic insulators, such spectral charge enables one to describe the topology of every band without regard for Fermi level [5]. The simulated Wannier configuration, i.e., the spectral charge profile of Band 1 is provided in the right panel of Fig. 1a. The area of the circles represents the charge quantity of each plaquette and the color maps are the charges of each unit-cell modulo an integer; see results of Bands 2 and 3 in the Supplementary materials Note 3. We choose the lower-left unit-cell of the PC to show the calculation of the charge quantity in one unit-cell consisting of different plaquettes. The central bulk plaquette hosts 1 charge while the upper-right bulk plaquette contributes 0.25 charge since it is portioned out equally by four adjacent unit-cells. Likewise, two edge plaquettes provide 0.5 charge each and the corner plaquette contributes 1 charge.
Defined as charge of the corner unit-cell modulo an integer, topological corner anomaly is obtained by summing all plaquette charges contained in the corner unit-cell: \((1 + 1 + 0.5 \times 2 + 0.25)\) \(\mod 1 = 0.25\). The charge quantities of all four corner unit-cells are expected to be identical due to the \(C_4\) rotation symmetry. Likewise, each edge unit-cell hosts 0.5 charge and each bulk unit-cell hosts integer charge due to the translation symmetry.

We next demonstrate the experimental verifications of the topological corner anomaly of Phase A. We measured LDOS spectra of the bulk, edge and corner plaquettes in the lower-left unit-cell and provide the results in Fig. 1c, where the experimental results are in good agreement with the simulations in Fig. 1a (see the Supplementary materials Note 5 for measurement methods). By integrating the LDOS spectra over their respective spectrum bands, i.e., Bands 1, 2 and 3, the measured spectral charges of different plaquettes are presented as three pie charts alongside the LDOS spectra. From both experimental and simulated charges, we conclude that the charge proportion of three spectrum bands for the bulk, edge and corner plaquettes is \(1 : 2 : 1, 1 : 1 : 1\) and \(1 : 0 : 1\) in turn. We then sum the plaquette charges of the whole unit-cell to obtain the topological corner anomaly manifesting as fractional spectral charges (Fig. 1b). Both experimental and simulated results manifest that an additional charge arises from the corner. The fractional spectral charge is quantized to nearly 0.25 for both Bands 1 and 3, and nearly 0.5 for Band 2. It should be noted that Bands 1 and 3 host the same charge quantity of 0.25 because they correspond to the orbitals \(s\) and \(d\), respectively. Band 2 is twofold degenerated and corresponds to the orbitals \(p_x/p_y\), giving rise to the charge of 0.25 \(\times 2 = 0.5\).

By tuning the relative strength between the inter- and intra-plaquette couplings, the PC in Phases A can switch into Phase B; see the Supplementary materials Note 2 for the detailed phase transition. The PC in Phase B and the simulated Wannier configurations are shown in the left and right panel of Fig. 1d, respectively. The lower-left corner unit-cell of the PC (inset) contains two edge plaquettes and two bulk plaquettes, and each bulk plaquette is portioned equally by two adjacent unit-cells. Hence, the unit-cell charge is calculated as \((1 \times 2 + 0.5 \times 2)\) \(\mod 1 = 0\). As with Phase A, we can measure the LDOS spectra of different plaquettes in Phase B, and then obtain the unit-cell charges of Bands 1–3 by summing the measured charges of four plaquette together (see the Supplementary materials Note 3 for details). Consistent with the simulations, the measured spectral charge in Fig. 1e is nearly 0 modulo 1 for all three bands.

Comparing the acoustic Wannier configurations in Phase A and Phase B, we observe that Phase A indicates pronounced charge values for the corner unit-cells but zero charge for the edge ones. The fractional spectral charges which correspond to four corner-localized SWFs indicate the existence of corner modes, manifesting as topological corner anomaly. On the contrary, all unit-cells of the PC corresponding to Phase B host zero charge, indicating that the Wannier configuration is in trivial phase and does not support topological corner-localized modes. Secondly, the fractional spectral charge revealing topological corner anomaly of Phase A can be excited in all three spectrum bands. The proportion of this corner anomaly is \(0.25 : 0.5 : 0.25 = 1 : 2 : 1\) for three bands, which is consistent with the fact that the orbitals \(p_x/p_y\) correspond to two degenerated bands while the orbitals \(s\) and \(d\) correspond to one band. In fact, the proposed topological corner anomaly appears because the boundary (corner and edge) plaquettes in the PCs host different eigenmode spectra from the bulk ones, giving rise to anomalous spectrum distributions of the acoustic energy (the Supplementary materials Note 1). This intrinsic signature is unrelated to the corner mode frequency and can be detected from all three bulk bands for both Phases A and B (see the Supplementary materials Note 3 for calculations). We would like to emphasize that in the proposed PCs of Phases A and B, the outer boundaries are the natural results of periodically arranging the unit-cells without boundary compensations such as thin air layers [11] or auxiliary resonators [16]. Therefore, Phases A and B only host crystalline symmetries. The proposed spectral charge characterization thus offers a real-space manner to capture HOTIs for acoustic systems without chiral or other intrinsic symmetries, even when the corner modes are merged into the bulk bands.

With acoustic Wannier configurations revealing corner signatures studied, spectrally identified corner modes which lie within the band gap are further expected. To achieve this goal, we separately tune the coupling strength of the PCs in Phases A and B (the Supplementary materials Note 4), and then combine the PCs in two phases together to obtain combined systems where in-gap corner modes are expected to localize at the intersection of two interfaces.

As shown in Fig. 2a, a combined system named Combination 1 is constructed by embedding the PC of Phase A into that of Phase B. The combined Wannier configuration is illustrated in the left panel of Fig. 2b, where the SWFs of the inner configuration are labelled in red and the SWFs of the outer one are labelled in blue. Within the bulk region, the orbitals carrying integer charges make no contributions to the boundary charges. Hence, the crystalline topology is determined by the interface charge signatures. Along the interface, the red orbitals interleave with the blue ones, and each interface orbital hosts half-integer charges, leading to topological interface modes [13]. The interface Wannier orbitals are terminated at the corner, i.e., the intersection of the two interfaces, with the corner orbital containing only one SWF of 0.25 charge. Hence, the missing charge quantity of this incomplete corner orbital is \(1 – 0.25 = 0.75\). Such pictorially determined charge coincides well with the simulated results in the right panel of Fig. 2b where the missing charge representing topological corner anomaly of Band 1 is also nearly 0.75 (see the Supplementary materials Note 4 for details). The simulations of normalized LDOS in Fig. 2c indicate that the corner modes at the frequency of 1.92 kHz are in the bandgap, and the simulated acoustic field (Fig. 2a) shows that these corner modes are highly localized.

Next, we construct another type of corner modes by exchanging the above inner and outer Wannier configurations. The resultant system is denoted as Combination 2 as illustrated in Fig. 2d, where Phase B is embedded into Phase A. In Fig. 2c, we provide the simulated normalized LDOS of the bulk, edge and corner regions and the measured LDOS of the corner cavities. The simulation and observation together show that the corner modes appear in the bandgap at 1.1 kHz (red arrows). The corresponding field intensity distribution for the whole PC is simulated in Fig. 2d, indicating the existence of corner modes with \(C_4\) rotation symmetry. In experiments, we scanned a square area around the lower-right corner with 36 cavities and present the measured acoustic field at 1.1 kHz in the upper inset of Fig. 2d. Both simulated and experimental results indicate that the acoustic energy is highly localized at the corner cavities. Such field distribution feature corresponds to the combined Wannier configurations shown in the left panel of Fig. 2e, where the combined corner orbital contains three SWFs which render 0.25 charge each. The missing charge is \(1 – 0.25 \times 3 = 0.25\). Above pictorially determined charge is verified by simulating the interface local charge profiles of Combination 2 (right panel, Fig. 2e). In this combination, the topological corner anomaly originates from the fractional corner orbitals of the outer configuration, i.e., Phase A, and the acoustic energy is mainly at the outside of each corner.

In addition, by changing the relative coupling strengths of PCs of the inner and outer Wannier configurations, we obtain another two different combinations of the PCs, namely Combination 3.
and Combination 4. These two combinations also support topological in-gap corner modes and the associated simulations are provided in the Supplementary materials Note 4. For all four combinations, both the ordinary and the exotic phases can serve as the cladding layers in the combined PCs, providing an anomalous scheme to engineering in-gap corner modes. Such scheme is simply based on splicing unit-cell building blocks and is robust to the possible experimental deformations or disruptions on the boundary, which is different from previous approaches of applying fine-tuned boundary compensations [11,16] or certain on-site potentials [6].

In conclusion, we experimentally demonstrate the acoustic Wannier configurations by measuring the spectral charge profiles of SWFs in real space. Fractional spectral charges in corner unit-cells are measured to represent the topological indices of HOTIs as real-space observables. By proposing two different Wannier configurations, we provide a complete discussion on the possible locations of the SWFs in 2D square lattice, which is different from previous approaches of applying fine-tuned boundary compensations [11,16] or certain on-site potentials [6].

It is important to note that the optical in-gap corner modes and the associated simulations are provided in the Supplementary materials Note 4. For all four combinations, both the ordinary and the exotic phases can serve as the cladding layers in the combined PCs, providing an anomalous scheme to engineering in-gap corner modes. Such scheme is simply based on splicing unit-cell building blocks and is robust to the possible experimental deformations or disruptions on the boundary, which is different from previous approaches of applying fine-tuned boundary compensations [11,16] or certain on-site potentials [6].

In conclusion, we experimentally demonstrate the acoustic Wannier configurations by measuring the spectral charge profiles of SWFs in real space. Fractional spectral charges in corner unit-cells are measured to represent the topological indices of HOTIs as real-space observables. By proposing two different Wannier configurations, we provide a complete discussion on the possible locations of the SWFs in 2D square lattice, which is different from previous approaches of applying fine-tuned boundary compensations [11,16] or certain on-site potentials [6].

Conflict of interest

The authors declare that they have no conflict of interest.
Acknowledgments

The authors thank Dr. Tuo Liu for fruitful discussions. This work was supported by the Key-Area Research and Development Program of Guangdong Province (2020B010190002), the National Natural Science Foundation of China (11890701, 11874383, 12104480, 11974005, and 12222405), the National Key R&D Program of China (2018YFA0305800), and the IACAS Frontier Exploration Project (QYTS202110).

Author contributions

Peng Zhang and Han Jia conceived the idea. Peng Zhang performed the numerical simulations and theoretical analyses. Peng Zhang and Xinghang Yang carried out all measurements. Zhengyou Liu and Jun Yang supervised the whole project. Peng Zhang, Han Jia, and Jiuyang Lu wrote the manuscript. All authors contributed to discussions of the experimental results and the manuscript.

Appendix A. Supplementary materials

Supplementary materials to this short communication can be found online at https://doi.org/10.1016/j.scib.2023.03.015.

References


Peng Zhang is a Ph.D. candidate at the Institute of Acoustics, Chinese Academy of Sciences. He received his B.S. and Ph.D. degrees from Wuhan University in 2018. His research interest focuses on acoustic topological metamaterials, piezoelectric phononic crystals and advance manipulation on acoustic topological states.

Han Jia received his Ph.D. degree from Wuhan University in 2012. He is currently a professor at the Institute of Acoustics, Chinese Academy of Sciences (IACAS). His research interest includes acoustic metamaterials, transformation acoustics and acoustic functional sensors.

Jiuyang Lu is an associate professor at the School of Physics and Optoelectronics, South China University of Technology. He received his B.S. and Ph.D. degrees from Wuhan University. He is mainly interested in topological phonics and metamaterials, including the acoustic and elastic topological insulators, nodal point and nodal line semimetals and advanced acoustic functional materials.

Zhengyou Liu received his Ph.D. degree in 1993 from Wuhan University. Before he joined Wuhan University in 2001, he held the position of a professor at South China University of Technology. He is currently a professor at the School of Physics and Technology. His current research interest includes wave physics, phonic crystals and acoustic metamaterials, acoustic radiation force and particle manipulation, and topological acoustics.

Jun Yang received his Ph.D. degree from Nanjing university in 1996. He has been a Professor at IACAS since 2003. He is currently a Deputy Director of IACAS. His research interests include communication acoustics, 3D audio systems, acoustic signal processing, sound field control, nonlinear acoustics and acoustic metamaterials.