Optimal Power Allocation Algorithm for Radar Network Systems
Based on Low Probability of Intercept Optimization

Shi Chen-guang    Wang Fei    Zhou Jian-jiang*    Chen Jun
(Key Laboratory of Radar Imaging and Microwave Photonics, Ministry of Education,
Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China)

Abstract: A novel optimal power allocation algorithm for radar network systems is proposed for Low Probability
of Intercept (LPI) technology in modern electronic warfare. The algorithm is based on the LPI optimization. First,
the Schleher intercept factor for a radar network is derived, and then the Schleher intercept factor is minimized by
optimizing the transmission power allocation among netted radars in the network to guarantee target-tracking
performance. Furthermore, the Nonlinear Programming Genetic Algorithm (NPGA) is used to solve the resulting
nonconvex, nonlinear, and constrained optimization problem. Numerical simulation results show the effectiveness
of the proposed algorithm.

Key words: Radar network systems; Low Probability of Intercept (LPI); Schleher intercept factor; Target tracking;
Kullback-Leibler (KL) divergence

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1 Introduction

Distributed radar network system, which is often called as distributed Multiple-Input Multiple-Output (MIMO)
radar or statistical MIMO radar,
has received great attentions in recent years, for it can offer improved performance by employing signal and spatial diversities. In addition, radar network outperforms traditional monostatic radar in target detection, estimation accuracy, information extraction and Low Probability of Intercept (LPI)\textsuperscript{[3–3]}. Currently, system optimization for target detection and information extraction in radar network has been under long and intensive research. In Ref. [4], Fisher et al. propose the concept of statistical MIMO radar for the first time, where the fundamental differences between statistical MIMO and other radar array systems are analyzed. Friedlander presents an algorithm to maximize the total power of the sampled echoes in power
allocation\cite{10}. Tang et al. consider the problem of optimal waveform design for MIMO radar in colored noise\cite{11}. In Ref.\cite{7}, Yang and Blum investigate two radar waveform design problems with constraints on waveform power: the first one is to maximize the Mutual Information (MI) between the target impulse response and the reflected waveform, and the second one is to minimize the value of the Minimum Mean-Square Error (MMSE) in estimating the target impulse response. The authors of Ref.\cite{8} develop a novel two-stage approach to optimize the waveforms of an adaptive MIMO radar, and it is demonstrated that the new approach can provide great performance improvement in terms of target response extraction and radar scene generation for real-time practical scenarios. Song et al. formulate the interaction between a smart target and a smart MIMO radar from a game theory perspective, where the unilateral, hierarchical, and symmetric games are studied\cite{9}. Niu et al. in Ref.\cite{10} propose localization and tracking algorithms for a non-coherent MIMO radar.

The research on power allocation for radar network has received increasing impetus\cite{11-16}. The authors in Refs.\cite{11-13} concentrate on the optimal power allocation in distributed multiple-radar architectures, where the Cramer-Rao Bound (CRB) is utilized as an optimization metric. In Ref.\cite{14}, Chavali and Nehorai compute the posterior CRB on the estimates of the target state and the channel state, and use it as an optimization criterion for power allocation and antenna selection in a cognitive radar network. Song et al. present three power allocation criteria integrating the propagation losses into the MIMO radar signal model: maximizing the MI, minimizing the MMSE, and maximizing the echo energy\cite{15}. Further, Song et al. in Ref.\cite{16} optimize the detector performance for distributed MIMO radar with optimal power allocation to enhance target detection.

Based on the above discussions, most cases focus on the problem of performance optimization for radar network such as target localization and detection. System performance enhancement in target tracking can be achieved with an increase in either the number of active radars or the transmission power. In practice, for a fixed radar network, the achievable target tracking performance would beyond a predefined threshold with full transmission power allocation, where LPI operation is required in modern electronic warfare. However, up to now, we have not seen any studies on LPI optimization for radar network, which is playing an increasing important role in modern battlefield\cite{17-20}. This motivates us to consider the matter for the first time.

This paper extends the result in Ref.\cite{16} and presents a novel optimal power allocation algorithm based on LPI optimization for radar network systems, which minimizes Schleher intercept factor by optimizing transmission power allocation among netted radars on the guarantee of target tracking performance. Due to the lack of analytical closed-form expression for Receiver Operation Characteristics (ROC), we employ Kullback-Leibler (KL) divergence as the metric for target detection performance. Subsequently, the Nonlinear Programming based Genetic Algorithm (NPGA) is employed to solve the resulting nonconvex, nonlinear and constrained optimization problem. As shown later, the proposed algorithm can offer significant LPI performance enhancement for radar network. To our best of authors’ knowledge, no literature discussing the LPI optimization problem for radar network systems was prior to this work.

The remainder of this paper is organized as follows. Section 2 introduces the radar network system model and the optimal detector. In Section 3, we first derive Schleher intercept factor for radar network, and then formulate the problem of optimal power allocation for radar network based on LPI optimization, where the resulting nonconvex, nonlinear and constrained problem is solved through NPGA. Numerical simulation results are provided in Section 4 to demonstrate the effectiveness of the proposed algorithm. Finally, Section 5 concludes this paper.
2 System Model and Preliminaries

2.1 Radar network SNR equation

We consider an \( M \times N \) radar network system with \( M \) transmitters and \( N \) receivers, which can be broken down into \( M \times N \) transmitter-receiver pairs each with a bistatic component contributing to the entirety of the radar network Signal-to-Noise Ratio (SNR). As depicted in Fig. 1, all the radars in the network have acquired and are tracking the target with their directional radar beams. The netted radars transmit orthogonal waveforms but receive and process all these echoes that are reflected from the target, and send the estimates to the data fusion center with high speed data link. We suppose that the network system has a common precise knowledge of space and time.

Herein, for a radar network, orthogonal poly-phase codes are utilized in the system, which have a large main lobe-to-side lobe ratio. These codes have a more complicated signal structure, making it harder to be intercepted and detected by a hostile intercept receiver.

Assuming that each transmitter-receiver pair combination in the network is the same, the radar network SNR can be calculated as introduced in Ref. [1]:

\[
\text{SNR}_N = \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{E_{ii}G_{ij}\sigma_i \lambda^2}{(4\pi)^3 k T_o B_i F_i G_{ij} R_{ij}^2 L_i} \tag{1}
\]

where \( E_{ii} \) is the \( i \)-th transmitter power, \( G_i \) is transmitting antenna gain, \( G_{ij} \) is receiving antenna gain, \( \sigma_i \) is the Radar Cross Section (RCS) of the target, \( \lambda \) is transmitted wavelength, \( k \) is Boltzmann’s constant, \( T_o \) is the receiving system noise temperature, \( B_i \) is the bandwidth of the matched filter for transmitted waveform, \( F_i \) is the noise factor for receiver, \( L_i \) is system loss, \( R_{ii} \) is the distance from the \( i \)-th transmitter to the target, and \( R_{ij} \) is the distance from the target to the \( j \)-th receiver.

2.2 Radar network signal model

According to the discussions in Ref. [16], the path gain contains the target reflection coefficient \( g_{ij} \) and the propagation loss factor \( p_{ij} \). Based on the central limit theorem, \( g_{ij} \sim \mathcal{CN}(0, R_{ij}) \), where \( g_{ij} \) denotes the target reflection gain between the radar \( i \) and radar \( j \). The propagation loss factor \( p_{ij} \) is a function of radar antenna gain and waveform propagation distance, which is expressed as:

\[
p_{ij} = \sqrt{\frac{G_i G_{ij}}{R_{ii}R_{ij}}} \tag{2}
\]

It is supposed that the transmitted waveform of the \( i \)-th netted radar is \( \sqrt{E_{ij}}s_i(t) \), and then the collected signals at the \( j \)-th receiver from a single point target can be written as:

\[
y_j(t) = \sum_{i=1}^{M} p_{ij} g_j \sqrt{E_{ij}} s_i(t - \tau_{ij}) + n_j(t) \tag{3}
\]

where \( \int |s_j(t)|^2 dt = 1 \), \( \tau_{ij} \) represents the time delay, \( n_j(t) \) denotes the noise at receiver \( j \), and the Doppler effect is negligible. At the \( j \)-th receiver, the received signal is matched filtered by time response \( s_j^*(t) \), and the output signal can be expressed as:

\[
\tilde{y}_{jk} (t) = \int y_j(t) s_j^* (t - \tau) \, d\tau = p_{jk} g_{jk} \sqrt{E_{jk}} \int s_j(t - \tau_{jk}) s_j^* (t - \tau) \, d\tau + \tilde{n}_{jk} (t) \tag{4}
\]

where \( \tilde{n}_{jk} (t) = \int n_j(\tau) s_j^* (\tau - t) \, d\tau \), and \( \int s_j(\tau) s_j^* (\tau + \tau) \, d\tau = 0 \) for \( k \neq j \).

The discrete time signal for the \( j \)-th receiver can be rewritten as:

\[
r_{jk} = \tilde{y}_{jk} (\tau_{jk}) = p_{jk} g_{jk} \sqrt{E_{jk}} + \theta_{jk} \tag{5}
\]

where \( r_{jk} \) is the output of the matched filter at the receiver \( j \) sampled at \( \tau_{jk} \), \( \theta_{jk} = \tilde{n}_{jk} (\tau_{jk}) \), and \( \theta_{jk} \sim \mathcal{CN}(0, R_{jk}) \). As mentioned before, we assume that all the netted radars have acquired and are tracking the target with their pencil beams, and they transmit orthogonal waveforms while receive and process all these echoes that are reflected from the target. In this way, we can obtain the time delay \( \tau_{jk} \).

Fig. 1  Example of a radar network system
2.3 The optimal detector

With all the received signals, the target detection for radar network system leads to a binary hypothesis testing problem:

\[ H_0 : r_i = \theta_j \]
\[ H_1 : r_i = p_{ij} \theta_j \sqrt{E_{ii} + \theta_j} \]  

(6)

where \( 1 \leq i \leq M, 1 \leq j \leq N \). As introduced in Ref. [16], the underlying detection problem can be equivalently rewritten as follows:

\[ H_0 : r_{ij} \sim \mathcal{CN}(0,R_0) \]
\[ H_1 : r_{ij} \sim \mathcal{CN}(0,R_0 + E_t R_i F_i L_i) \]

(7)

Then, we have the optimal detector as:

\[ H_0 : T \triangleq \sum_{i=1}^{M} \sum_{j=1}^{N} |r_{ij}|^2 \frac{2E_{ii}p_{ij}^2}{R_0 + E_t R_i F_i L_i} < \delta \]
\[ H_1 : T \triangleq \sum_{i=1}^{M} \sum_{j=1}^{N} |r_{ij}|^2 \frac{2E_{ii}p_{ij}^2}{R_0 + E_t R_i F_i L_i} > \delta \]

(8)

where \( \delta \) denotes the detection threshold.

3 Problem Formulation

In this section, we aim to obtain the optimal LPI performance for radar network system by judiciously designing the transmission power allocation among netted radars in the network. We first derive Schleher intercept factor for radar network system, and then formulate the optimal power allocation problem based on LPI optimization. For a predefined threshold of target tracking performance, Schleher intercept factor is minimized by optimizing transmission power allocation among netted radars in the network to support the LPI performance improvement. It is indicated in Ref. [21] that the analytical closed-form expression for Receiver Operation Curve (ROC) does not exist. As such, we resort to KL-divergence. In the following, we employ the NPGA to solve the resulting nonconvex, nonlinear, and constrained problem.

3.1 Schleher intercept factor for radar network

For a radar network, we suppose that all the signals can be separately distinguished at every radar node in the network. Assuming that \( R_N^2 \triangleq R_i R_j \), and Eq. (1) can be rewritten as:

\[ \text{SNR}_N = C_R N \frac{E_t}{R_N} \]  

(9)

where

\[ C_R = \frac{G_i G_i \sigma^2}{(4\pi)^3 k T_i B_i F_i L_i} \]

and \( E_t \) is the total transmitting power of radar network system.

Note that when \( M = N = 1 \), we can obtain the traditional monostatic case:

\[ \text{SNR}_M = C_M \frac{E_t}{R_M^2} \]  

(11)

where \( R_M \) is the distance between the monostatic radar and the target.

While for intercept receiver, the SNR equation is:

\[ \text{SNR}_I = \frac{E_t C_I}{R_I^2} \]  

(12)

where

\[ C_I = \frac{G_i G_i \lambda^2}{(4\pi)^3 k T_i B_i F_i L_i} \]  

(13)

\[ \text{SNR}_I \] is the SNR at the interceptor signal processor input, \( G_i \) is the gain of the radar’s transmitting antenna in the direction of the intercept receiver, \( G_i \) is the gain of the intercept receiver’s antenna, \( F_i \) is the interceptor noise factor, \( B_i \) is the bandwidth of the interceptor, \( R_I \) is the range from radar network to the intercept receiver, and \( L_i \) refers to the losses from the radar antenna to the receiver. Assuming that the interceptor detects the radar emission from the main lobe, then \( G_i = G_i \).

Here, Schleher intercept factor is employed to evaluate LPI performance for radar network\[28\]. The definition of intercept factor can be calculated as follows:

\[ \alpha = \frac{R_I}{R_0} \]  

(14)

where \( R_0 \) is the detection range of radar, and \( R_I \) is the intercept range of intercept receiver, as illustrated in Fig. 2.

Based on the definition of Schleher intercept factor
factor, if $\alpha > 1$, radar can be detected by the interceptor. While if $\alpha \leq 1$, radar can detect the target while the interceptor can not detect the radar. Therefore, radar can meet LPI performance when $\alpha \leq 1$. Moreover, minimization of Schleher intercept factor results in better LPI performance for radar network system.

According to the definition of Schleher intercept factor, we can derive the intercept factor for radar network:

$$\alpha_N = \frac{R_I}{R_N} = \sqrt{\frac{E[C_i^2 | \text{SNR}_N]}{C_r | \text{SNR}_i^2}}$$  \hspace{1cm} (15)

and the intercept factor for conventional monostatic case as:

$$\alpha_M = \frac{R_I}{R_M} = \sqrt{\frac{E[C_i^2 | \text{SNR}_M]}{C_r | \text{SNR}_i^2}}$$  \hspace{1cm} (16)

For simplicity, the intercept factor $\alpha_M$ is normalized to be 1 when the monostatic radar transmits the maximum power $E_{\text{tot}}$, and $\text{SNR}_N = \text{SNR}_M$. Therefore, when the transmission power is $E_i$, Eq. (15) can be rewritten as:

$$\alpha_N = \frac{\alpha_M}{\sqrt{N}} = \sqrt{\frac{E_i}{E_{\text{tot}} N}}$$  \hspace{1cm} (17)

Obviously, it is observed that Eq. (17) will decrease correspondingly, with the increase of the number of radar receivers $N$ in the network and the decrease of the total transmission power $E_i$ of radar network.

3.2 LPI optimization based optimal power allocation algorithm

It is introduced in Ref. [21] that KL-divergence $D(p_0 || p_1)$ measures the distance between two probability density functions (pdf) $p_0$ and $p_1$. Let $D(f(r | H_0) || f(r | H_1))$ represent KL-divergence between $H_0$ and $H_1$, where $f(r | H_0)$ and $f(r | H_1)$ are the pdfs of $r$ under hypotheses $H_0$ and $H_1$. For any fixed value of the probability of false alarm $P_a$,

$$D[f(r | H_0) || f(r | H_1)] = \lim_{N \to \infty} \frac{1}{N} \log(1 - P_a)$$  \hspace{1cm} (18)

From Eq. (18), we can see that for any fixed $P_a$, the maximization of KL-divergence $D[f(r | H_0) || f(r | H_1)]$ leads to an asymptotic maximization of the probability of detection $P_d$. With the derivation in Ref. [16], we can obtain:

$$D_{\text{net}} \triangleq D[f(r | H_0) || f(r | H_1)] = \int f(r | H_0) \log \frac{f(r | H_0)}{f(r | H_1)} \ dr$$

$$= \sum_{i=1}^{M} \sum_{j=1}^{N} \log \left(1 + \frac{E_i R_y R_y^{-1} P_0^2}{1} \right)$$

$$+ \left(1 + E_i R_y R_y^{-1} P_0^2 \right)^{-1} - 1$$  \hspace{1cm} (19)

Based on the discussion in Ref. [21], the maximization of KL-divergence between the two distributions of the hypotheses means better target tracking performance for radar network. Intuitively, maximization of KL-divergence Eq. (18) means allocating more transmission power, which would make the radar network system more vulnerable in modern electronic warfare.

Herein, we look into the optimal power allocation for radar network, whose purpose is to minimize Schleher intercept factor for a predefined KL-divergence threshold such that the LPI performance is met. Consequently, the underlying power allocation problem for radar network can be formulated as:

$$\min_{E} \alpha_N,$$

$$\text{s.t.: } D_{\text{net}} \geq D^{th},$$

$$\sum_{i=1}^{M} E_{i,t} \leq E_{\text{tot}}^{\text{max}},$$

$$0 < E_{i,t} \leq E_{i,t}^{\text{max}}, \forall i$$  \hspace{1cm} (20)

where $E = [E_{1,t}, E_{2,t}, \ldots, E_{M,t}]^T$ is the transmitting power of radar network, $D^{th}$ is the KL-divergence threshold for target detection, $E_{\text{tot}}$ is the maximum total transmission power of radar network, and $E_{i,t}^{\text{max}}, \forall i$ is the maximum transmission power of the corresponding netted radar node.

3.3 The nonlinear programming based genetic algorithm

In this paper, we utilize the NPGA to seek the optimal solution to the nonconvex, nonlinear, and constrained problem Eq. (20).

The NPGA procedure is illustrated in Fig. 3, where the population initialization module is utilized to initialize the population according to the resulting problem, while the calculating fitness value module is to calculate the fitness values of individuals in the population. Selection,
crossover and mutation are employed to seek the optimal solution, where $N$ is a constant. If the evolution is $N$’s multiples, we can use the nonlinear programming approach to accelerate the convergence speed. The NPGA has a good performance on the convergence speed and the result, and it improves the searching performance of ordinary genetic algorithm.

So far, we have completed the derivation of Schleher intercept factor for radar network and the formulation of target detection based optimal power allocation problem. In the following, several numerical simulations are provided to confirm the effectiveness of the presented algorithm for radar network system.

4 Numerical Simulations

In this section, we provide some numerical simulations to verify the proposed optimal power allocation algorithm as Eq. (20). Throughout this section, we suppose $E_{\text{tot}}^{\max} = \sum_{i=1}^{M} E_{ti} = 24$ kW, $E_{ti}^{\max} = 6$ kW, $G_i = G_t = 30$ dB, $R_g = 10^{-10}$, and $R_g = 1$. The SNR is set to be 13 dB. The traditional monostatic radar can detect the target whose RCS is 0.05 m$^2$ in the distance 106.1 km by transmitting the maximum power $E_{\text{tot}} = 24$ kW, where the intercept factor is normalized to be 1 for simplicity.

4.1 LPI performance analysis

Fig. 4 shows the KL-divergence versus Schleher intercept factor for different radar network architectures, which is conducted $10^6$ Monte Carlo trials. It can be observed that as Schleher intercept factor increases from $\alpha_N = 0$ to $\alpha_N = 2$, the available KL-divergence is increased. This is due to the fact that as the intercept factor increases, more power would be transmitted, which makes the available KL-divergence increase correspondingly as theoretically proved in Eq. (19). In addition, one can see from Fig. 4 that with the same KL-divergence threshold, Schleher intercept factor can be significantly reduced as the number of transmitters and receivers in the network increases. Therefore, increasing the number of the netted radars can effectively improve the LPI performance for radar network. This confirms the LPI benefits of the radar network architecture with more netted radars.

In Fig. 5, we illustrate the KL-divergence versus Schleher intercept factor for different target scattering intensities with $M=N=4$. It is depicted that as the target scattering intensity increases from $R_s = 1$ to $R_s = 10$, the available KL-divergence is significantly increased. This is because the radar network system can detect the target with large scattering intensity easily with high $P_d$ and low $P_f$.

4.2 Target tracking with the proposed algorithm

In this subsection, we consider a $4\times4$ radar network system ($M=N=4$) in the simulation, which is widely deployed in modern battlefield. The target detection threshold $D^h$ can be calculated in the condition that the transmitting power of each radar is 6 kW in the distance of 150 km between the radar network and the target. As mentioned before, we assume that the intercept receiver is carried by the single target. It is illustrated in Fig. 6 that the netted radars in the network are spatially distributed in the surveillance area.

We employ the Particle Filtering (PF) method to track a single target, where 5000 particles are utilized to estimate the target state. Without loss
of generality, we set Radar 1 as the data fusion centre, and capitalize the weighted average approach to obtain the estimated target state. Fig. 7 depicts one realization of the target trajectory for 50 s, and the tracking interval is chosen to be 1 s. Fig. 8 illustrates the distance changing curve between the netted radars and the target in the process of target tracking.

To obtain the optimal transmission power allocation for radar network, we employ NPGA to solve Eq. (20), where the population size is initialized to be 100, the crossover probability is 0.6 and the mutation probability is 0.01. The population evolves 10 generations. Fig. 9 shows the transmission power of netted radars utilizing the proposed algorithm in the tracking process.

The advantage of the proposed algorithm is demonstrated in Fig. 10. The traditional monostatic radar transmits 24 kW constantly, while the ordinary radar network has a constant sum of

![KL-divergence versus Schleher intercept factor](image1)

![KL-divergence versus Schleher intercept factor for different radar network architectures](image2)

![KL-divergence versus Schleher intercept factor for different target scattering intensity with M=N=4](image3)

![The radar network system configuration in two dimensions](image4)

![Target tracking scenario](image5)

![The distances between the netted radars and the target](image6)
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Fig. 9  The transmission power of netted radars utilizing LPI optimization based optimal power allocation in the tracking process

transmitted power 24 kW and each radar node transmits uniform power. One can observe obviously that Schleher intercept factor for radar network employing the LPI optimization based optimal power allocation algorithm is strictly smaller than that of traditional monostatic radar and ordinary radar network across the whole region, which further confirms the LPI improvement by exploiting the proposed algorithm in radar network to defend against passive intercept receiver.

4.3 Discussion

According to the Figs. 4–10, we summarize the following conclusions for radar network.

(1) From Fig. 4 and Fig. 5, we can see that as the predefined threshold of target detection increases, more transmission power would be allocated for radar network to meet the detection performance, while the intercept factor is increased subsequently, which is vulnerable and dangerous in modern electronic warfare. In other words, there is a tradeoff between target detection and LPI performance. It means that the LPI performance for radar network would be sacrificed with the target detection consideration.

(2) In the numerical simulations, we observe that the proposed optimal power allocation algorithm in Eq. (20) can be employed to enhance the LPI performance for radar network. Based on the netted radars’ spatial distribution with respect to the target, transmission power is allocated optimally among netted radars in the network system. As indicated in Fig. 10, one can see that exploiting the proposed algorithm can effectively improve the LPI performance for radar network.

5  Conclusions

In this paper, a novel LPI optimization based optimal power allocation algorithm is presented for radar network systems, where Schleher intercept factor is minimized by optimizing transmission power allocation among netted radars in the network on the guarantee of target tracking performance. Simulation results have been provided to demonstrate that the proposed algorithm is effective and valuable to enhance the LPI performance for radar network. Note that only a single target was considered in this paper. Nevertheless, it is convenient to be extended to multiple targets scenario, and the conclusions obtained in this study suggest that similar LPI benefits would be obtained for the multiple targets case. In the future work, we intend to investigate other optimization criteria to improve LPI performance for radar network systems.

References

Shi Chen-guang (1989–) was born in Luoyang, China. He received the B.S. degree from Nanjing University of Aeronautics and Astronautics (NUAA) in 2012, and he is currently working toward his Ph.D. degree in NUAA. His main research interests include aircraft radio frequency stealth and radar signal processing.

E-mail: scg_space@163.com

Zhou Jian-jiang (1962–) was born in Nantong, China. He received the M.S. degree and the Ph.D. degree from Nanjing University of Aeronautics and Astronautics (NUAA) in 1988 and 2001 respectively, and then became a professor in NUAA. His main research interests include aircraft radio frequency stealth, radar signal processing, and array signal processing.

E-mail: zjjee@nuaa.edu.cn