

# Adaptive neural control based on HGO for hypersonic flight vehicles

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**Abstract** This paper describes the design of adaptive neural controller for the longitudinal dynamics of a generic hypersonic flight vehicle (HFV) which are decomposed into two functional systems, namely the altitude subsystem and the velocity subsystem. For each subsystem, one adaptive neural controller is investigated based on the normal output-feedback formulation. For the altitude subsystem, the high gain observer (HGO) is taken to estimate the unknown newly defined states. Only one neural network (NN) is employed to approximate the lumped uncertain system nonlinearity during the controller design which is considerably simpler than the ones based on back-stepping scheme with the strict-feedback form. The Lyapunov stability of the NN weights and filtered tracking error are guaranteed in the semiglobal sense. Numerical simulation study of step response demonstrates the effectiveness of the proposed strategy in spite of system uncertainty.

**Keywords** adaptive neural control, hypersonic flight vehicle, high gain observer, output-feedback

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## 1 Introduction

Hypersonic flight vehicles are intended to present a reliable and cost efficient way to access space even to launch small satellites into low earth orbit (LEO) [1]. The recent success of NASA's X-43A experimental airplane in flight testing has confirmed the feasibility of this technology. However, hypersonic flight vehicles have a highly nonlinear dynamics and because of their design and flight conditions of high altitudes and high Mach numbers, they are extremely sensitive to changes in atmospheric conditions as well as physical and aerodynamic parameters. As a result, modeling inaccuracies can have strong adverse effects on the performance of air vehicle's control systems. Controller designed for hypersonic aircraft must guarantee stability for the system and provide a satisfied control performance. Recently, feedback control strategy based on differential geometric nonlinear control theory had been proposed for the hypersonic aircraft, such as sliding mode control [2], robust control [3] and intelligent control [4, 5].

The sequential loop closure controller design [3, 6] with the decomposition of the equations into three functional subsystems is presented. The method followed the approach that combined robust adaptive dynamic inversion with back-stepping arguments to obtain a control architecture. As described in [5, 7],

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the altitude subsystem can be transformed into the strict-feedback form and the back-stepping design [8, 9] is adopted. Due to the system uncertainty, fuzzy logic system [5] and neural network [7] are employed because of their universal approximation ability. However, the determination of virtual control terms and their time derivatives during the back-stepping design requires tedious and complex analysis. More than one fuzzy or neural system is taken for approximation whose complexity increases as the order of the controlled system. For this problem, refs. [10, 11] provide a new way to deal with the strict-feedback system without back-stepping design. The main idea is to find a way to transform the strict-feedback system into some normal feedback form while the intelligent system is adopted to approximate the lumped uncertainty.

In this paper, we show that the altitude subsystem considering altitude, flight path angle, attack angle and pitch rate can be transformed into an output-feedback control problem and only one neural network is employed to approximate the lumped system nonlinearity. Moreover, we employ the high gain observer (HGO) to estimate the newly defined state variables for the adaptive neural controller design. The highlight is the simplicity of the controller design for the altitude subsystem of HFV. For the velocity subsystem, the output-feedback form is transformed and one adaptive neural controller is designed. It is proved that the signals involved are bounded via Lyapunov stability analysis.

This paper is organized as follows. Section 2 formulates the normal output-feedback form of the longitudinal dynamics of a generic hypersonic flight vehicle. The brief description of RBFN is explained in section 3. Section 4 presents the adaptive neural controller design and the stability analysis. The step response simulation is included in section 5. Section 6 presents several comments and final remarks.

## 2 Problem formulation

### 2.1 Hypersonic flight vehicle model

In this paper we consider the following longitudinal dynamic equations of a generic hypersonic aircraft [12] cruising at a Mach number of 15 and at an altitude of 110000 ft.

$$\dot{V} = \frac{T(V, \beta) \cos \alpha - D(V, \alpha)}{m} - \frac{\mu \sin \gamma}{r^2}, \quad (1)$$

$$\dot{h} = V \sin \gamma, \quad (2)$$

$$\dot{\gamma} = \frac{L(V, \alpha) + T(V, \beta) \sin \alpha}{mV} - \frac{(\mu - V^2 r) \cos \gamma}{Vr^2}, \quad (3)$$

$$\dot{\alpha} = q - \dot{\gamma}, \quad (4)$$

$$\dot{q} = \frac{M_{yy}(V, \alpha, q, \delta_e)}{I_{yy}}, \quad (5)$$

where  $V$  is the velocity,  $\gamma$  is the flight path angle,  $h$  is the altitude,  $\alpha$  is the attack angle,  $q$  is the pitch rate.  $T$ ,  $D$ ,  $L$  and  $M_{yy}$  represent thrust, drag, lift-force and pitching moment respectively.  $m$ ,  $I_{yy}$ ,  $\mu$  and  $r$  represent the mass of aircraft, moment of inertia about pitch axis, gravity constant and the radial distance from center of the earth.  $\delta_e$  is elevator deflection and  $\beta$  is the throttle setting.

$$L = \frac{1}{2} \rho V^2 S C_L,$$

$$D = \frac{1}{2} \rho V^2 S C_D,$$

$$T = \frac{1}{2} \rho V^2 S C_T,$$

$$M_{yy} = \frac{1}{2} \rho V^2 S \bar{c} [C_M(\alpha) + C_M(\delta_e) + C_M(q)],$$

$$r = h + R_E,$$

where  $\rho$  denotes the air density,  $S$  is the reference area,  $\bar{c}$  is the reference length and  $R_E$  is the radius of the Earth.  $C_x, x = L, D, T, M$  are the force and moment coefficients which are given by [2].

$$\begin{aligned} C_L &= 0.6203\alpha, \\ C_D &= 0.6450\alpha^2 + 0.0043378\alpha + 0.003772, \\ C_T &= \begin{cases} 0.02576\beta, & \text{if } \beta < 1, \\ 0.0224 + 0.00336\beta, & \text{otherwise,} \end{cases} \\ C_M(\alpha) &= -0.035\alpha^2 + 0.036617\alpha + 5.3261 \times 10^{-6}, \\ C_M(q) &= (q\bar{c}/2V) \times (-6.79\alpha^2 + 0.3015\alpha - 0.2289), \\ C_M(\delta e) &= 0.0292(\delta e - \alpha). \end{aligned}$$

We assume that engine dynamics i.e. the throttle setting are modeled by a second order system:

$$\ddot{\beta} = -2\xi\omega_n\dot{\beta} - \omega_n^2\beta + \omega_n^2\beta_c. \tag{6}$$

### 2.2 Output-feedback formulation

The control inputs are elevator deflection  $\delta_e$  and throttle setting  $\beta_c$ . By eqs. (1)–(5), the velocity is mainly related to throttle setting and the rate change of altitude is mainly related to the elevator deflection. So the dynamics can be decoupled into two functional subsystems. Given the tracking reference  $V_d$  and  $h_d$ , we design the velocity and altitude controller separately.

The velocity subsystem (1) can be rewritten as follows:

$$\begin{aligned} \dot{V} &= f_V(x_1, x_2, x_3, V) + g_V(x_1, x_2, V)\beta_c, \\ y_v &= V, \end{aligned} \tag{7}$$

where  $f_V = -(D/m + \mu \sin \gamma/r^2)$ ,  $g_V = \bar{q}S \times 0.02576 \cos \alpha/m$ ,  $\bar{q} = \frac{1}{2}\rho V^2$  if  $\beta_c < 1$ . Otherwise  $f_V = -(D/m + \mu \sin \gamma/r^2) + \bar{q}S \times 0.0224 \cos \alpha/m$ ,  $g_V = \bar{q}S \times 0.00336 \cos \alpha/m$ .

The control objective of the velocity subsystem (7) is to design an adaptive controller, which makes  $V \rightarrow V_d$ .

The tracking error of the altitude is defined as  $\tilde{h} = h - h_d$  and the flight path command is chosen as

$$\gamma_d = \arcsin \left[ \frac{-k_h(h - h_d) - k_I \int (h - h_d)dt + \dot{h}_d}{V} \right]. \tag{8}$$

From (2) and the definition of  $\tilde{h}$ , the corresponding dynamics for the altitude tracking error satisfy

$$\ddot{\tilde{h}} + k_h\dot{\tilde{h}} + k_I\tilde{h} = 0,$$

if  $k_h > 0$  and  $k_I > 0$  are chosen and the flight-path angle is controlled to follow  $\gamma_d$ , the altitude tracking error is regulated to zero exponentially.

According to the definition of flight path angle  $\gamma$ , attack angle  $\alpha$  and pitch angle  $\theta_p$ , we have  $\theta_p = \alpha + \gamma$ . Define  $X = [x_1, x_2, x_3]^T$ ,  $x_1 = \gamma$ ,  $x_2 = \theta_p$ ,  $x_3 = q$ .

The thrust term  $T(V, \beta) \sin \alpha$  in (3) can be neglected since it is generally much smaller than lift. Then the strict-feedback form equations of the altitude subsystem (2)–(5) are written as

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2, \\ \dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)x_3, \\ \dot{x}_3 &= f_3(x_1, x_2, x_3) + g_3(x_1, x_2)u, \\ y &= x_1, \end{aligned} \tag{9}$$

where  $f_1(x_1) = -(\mu - V^2r) \cos \gamma / (Vr^2) - (\bar{q}S \times 0.6203 / (mV)) \times \gamma$ ,  $g_1(x_1) = \bar{q}S \times 0.6203 / (mV)$ ,  $f_2(x_1, x_2) = 0$ ,  $g_2(x_1, x_2) = 1$ ,  $f_3(x_1, x_2, x_3) = \bar{q}S\bar{c}[C_M(\alpha) + C_M(q) - 0.0292\alpha] / I_{yy}$ ,  $g_3(x_1, x_2, x_3) = 0.0292\bar{q}S\bar{c} / I_{yy}$ ,  $u = \delta_e$ .

The control objective of system (9) is to design an adaptive controller, which makes  $\gamma \rightarrow \gamma_d$ , further  $h \rightarrow h_d$  and all the signals involved are bounded.

**Assumption 1.**  $f_1, f_3, f_V, g_1, g_3$  and  $g_V$  are unknown smooth functions. We assume there exist positive constants  $\bar{g}_{i1}, \bar{g}_{i2}, \bar{g}_{V1}$  and  $\bar{g}_{V2}$  such that  $\bar{g}_{i1} \geq g_i(\cdot) \geq \bar{g}_{i2} > 0$  and  $\bar{g}_{V1} \geq g_V \geq \bar{g}_{V2} > 0, i = 1, 3$ .

**Assumption 2.** There exist constants  $g_{1d}, g_{3d}$  and  $g_{Vd}$  such that  $g_{1d} \geq |\dot{g}_1|, g_{3d} \geq |\dot{g}_3|$  and  $g_{Vd} \geq |\dot{g}_V|$ .

In order to get the normal form of the altitude subsystem, the following transformation is conducted.

Let  $z_1 \triangleq y$  and  $z_2 \triangleq \dot{z}_1 = f_1 + g_1 x_2$ . The time derivative of  $z_2$  is derived as

$$\begin{aligned} \dot{z}_2 &= \frac{\partial f_1}{\partial x_1} \dot{x}_1 + \frac{\partial g_1}{\partial x_1} \dot{x}_1 x_2 + g_1 \dot{x}_2 \\ &= \left( \frac{\partial f_1}{\partial x_1} + \frac{\partial g_1}{\partial x_1} x_2 \right) (f_1 + g_1 x_2) + g_1 f_2 + g_1 g_2 x_3 \\ &\triangleq a_2(x_1, x_2) + b_2(x_1, x_2) x_3, \end{aligned} \quad (10)$$

where  $a_2(x_1, x_2) = \left( \frac{\partial f_1}{\partial x_1} + \frac{\partial g_1}{\partial x_1} x_2 \right) (f_1 + g_1 x_2) + g_1 f_2, b_2(x_1, x_2) = g_1 g_2 = g_1$ .

Again, let  $z_3 \triangleq \dot{z}_2 = a_2 + b_2 x_3$ . Then its time derivative is induced by

$$\begin{aligned} \dot{z}_3 &= \sum_{j=1}^2 \frac{\partial a_2}{\partial x_j} \dot{x}_j + \sum_{j=1}^2 \frac{\partial b_2}{\partial x_j} \dot{x}_j x_3 + b_2 \dot{x}_3 \\ &= \sum_{j=1}^2 \left( \frac{\partial a_2}{\partial x_j} + \frac{\partial b_2}{\partial x_j} x_3 \right) (f_j + g_j x_{j+1}) + b_2 (f_3 + g_3 u) \\ &\triangleq a_3(X) + b_3(X) u, \end{aligned} \quad (11)$$

where  $a_3(X) = \sum_{j=1}^2 \left( \frac{\partial a_2}{\partial x_j} + \frac{\partial b_2}{\partial x_j} x_3 \right) (f_j + g_j x_{j+1}) + b_2 f_3, b_3(X) = g_1 g_3$ .

As a result, the strict-feedback system (9) can be redescribed as the following normal form with respect to the newly defined state variables  $z_i$ 's:

$$\dot{z}_1 = z_2, \quad \dot{z}_2 = z_3, \quad \dot{z}_3 = a_3 + b_3 u, \quad y = z_1, \quad (12)$$

where  $a_3$  and  $b_3$  are functions of  $X$  and they are totally unknown.

### Remark 1.

1. The definition of  $f_V$  and  $g_V$  is up to the value of  $\beta_c$ . During the velocity controller design in subsection 4.2, we do not have to get the prior information of  $\beta_c$  and eq. (7) is for the explicit expression.

2. From Assumptions 1 and 2, it is noted that constants  $\bar{b}_3 > 0$  and  $b_{3d} > 0$  exist such that  $b_3 \geq \bar{b}_3$  and  $b_{3d} \geq |\dot{b}_3|, \forall X \in R^3$ .

3. From the definition of  $z_i$ 's, we know  $z_1 \triangleq y, z_2 \triangleq \dot{y}, z_3 \triangleq \ddot{y}$ .

4. From the definition of  $f_i, g_i, a_3$  and  $b_3$ , the state variables  $z_2$  and  $z_3$  are unknown.

## 3 Brief description of RBFN

In this paper, the RBFN is employed to approximate the unknown nonlinearity of the system.

$$\widehat{u}_{\text{RBFN}}(X_{\text{in}}) = \omega^T \varphi(X_{\text{in}}), \quad (13)$$

where  $X_{\text{in}} \in R^M$  is the input vector of the RBFN,  $\widehat{u}_{\text{RBFN}} \in R$  is the RBFN output,  $\omega \in R^{L_N}$  is the adjustable parameter vector,  $\varphi(\cdot) : R^M \rightarrow R^{L_N}$  is a nonlinear vector function of the inputs and  $L_N$  is the number of RBFs.

$$\varphi_i(X_{\text{in}}) = \exp \left( - \frac{|X_{\text{in}} - m_i|}{2\sigma_i^2} \right), \quad (14)$$

where  $m_i$  is an  $M$ -dimensional vector representing the center of the  $i$ th basis function and  $\sigma_i$  is the variance representing the spread of the basis function.

**Theorem 1.** For any given real continuous function  $u_{\text{RBFN}}^*$  on a compact set  $\Omega_{X_{\text{in}}} \in R^{n+1}$  and an arbitrary  $\varepsilon_{\text{re}} > 0$ , there exists one RBFN  $\widehat{u}$  in the form of (13) and an optimal parameter vector  $\omega^*$  such that

$$u_{\text{RBFN}}^*(X_{\text{in}}, \widehat{v}) = \widehat{u}_{\text{RBFN}}(X_{\text{in}}|\omega^*) + \zeta = \omega^{*\text{T}}\varphi(X_{\text{in}}) + \zeta, \tag{15}$$

$$\sup_{x_{\text{in}} \in \Omega_{x_{\text{in}}}} |\zeta| < \varepsilon_{\text{re}}, \tag{16}$$

where  $\varepsilon_{\text{re}} > 0$  denotes the supremum value of the reconstruction error  $\zeta$  that is inevitably generated.

## 4 Adaptive neural controller design and stability analysis

### 4.1 Adaptive controller for altitude subsystem

The following controller design is based on the scheme in [10, 13]. Vectors  $Y_d$ ,  $E$  and a filtered tracking error  $s$  are defined as follows:

$$Y_d = [y_d, \dot{y}_d, \ddot{y}_d]^{\text{T}}, \tag{17}$$

$$E = Z - Y_d, \tag{18}$$

$$s = \left( \frac{d}{dt} + \lambda \right)^2 E = [\Lambda^{\text{T}}1]E, \tag{19}$$

$$e = y - y_d = z_1 - y_d, \tag{20}$$

where  $\Lambda = [\lambda^2, 2\lambda]^{\text{T}}$  with  $\lambda > 0$ .

Since there exists complex nonlinear uncertainty of HFV, the  $z_i$ 's are incomputable. By employing the high gain observer [14], the estimation of  $Z = [z_1, z_2, z_3]^{\text{T}}$  is obtained as  $\hat{Z} = [z_1, \frac{\xi_2}{\varepsilon}, \frac{\xi_3}{\varepsilon^2}]^{\text{T}}$ .

The updating law of HGO is given as follows:

$$\begin{aligned} \dot{\xi}_1 &= \frac{\xi_2}{\varepsilon}, \\ \dot{\xi}_2 &= \frac{\xi_3}{\varepsilon}, \\ \dot{\xi}_3 &= \frac{-d_1\xi_3 - d_2\xi_2 - \xi_1 + y(t)}{\varepsilon}, \end{aligned} \tag{21}$$

where  $\varepsilon$  is a small design constant and parameters  $d_1$  and  $d_2$  are chosen such that the polynomial  $s^3 + d_1s^2 + d_2s + 1$  is Hurwitz and there exist positive constants  $h$  and  $t^*$  such that  $\forall t > t^*$  we have

$$|\hat{Z} - Z| \leq \varepsilon h. \tag{22}$$

The proof of conclusion (22) can be found in [14]. The estimations of  $E$  and  $s$  using (22) are denoted by

$$\widehat{E} = \widehat{Z} - Y_d, \tag{23}$$

$$\widehat{s} = [\Lambda^{\text{T}}1]\widehat{E}. \tag{24}$$

The derivative of  $s$  is obtained as

$$\dot{s} = [0 \ \Lambda^{\text{T}}]E + (y^{(3)} - y_d^{(3)}) = a_3 + b_3u - y_d^{(3)} + [0 \ \Lambda^{\text{T}}]E = a_3 + b_3u + \widehat{v} - [0 \ \Lambda^{\text{T}}]\tilde{E}, \tag{25}$$

where  $\widehat{v} = -y_d^{(3)} + [0 \ \Lambda^{\text{T}}]\widehat{E}$ ,  $\tilde{E} = \widehat{E} - E$ .

Note that the equality  $\tilde{E} = \tilde{Z}$  can be easily induced from (18) and (23),

$$\tilde{E} = \widehat{E} - E = \widehat{Z} - Z = \tilde{Z}. \tag{26}$$

Define

$$u_{\text{altitude}}^*(X_A) = \frac{a_3 + \widehat{v}}{b_3}, \quad X_A = [X^T, \widehat{v}]^T. \quad (27)$$

The  $u_{\text{altitude}}^*$  is approximated by the NN as

$$u_{\text{NN}} = \widehat{\omega}_A^T \varphi_A(X_A),$$

where  $\widehat{\omega}_A$  is the estimation of the optimal parameter vector  $\omega_A^*$ .

The update law for  $\widehat{\omega}_A$  is determined as

$$\dot{\widehat{\omega}}_A = \gamma_A (\widehat{s} \varphi_A(X_A) - \sigma_s(\widehat{\omega}_A) |\widehat{s}| \widehat{\omega}_A), \quad (28)$$

$$\sigma_s(\widehat{\omega}_A) = \begin{cases} c_A/\varepsilon_A, & \text{if } |\widehat{\omega}_A| > \varepsilon_A, \\ 0, & \text{otherwise,} \end{cases} \quad (29)$$

with  $\varepsilon_A$  being a design constant,  $\gamma_A$  the positive learning rate and  $|\varphi_A| \leq c_A$ . Then  $|\widehat{\omega}_A| \leq \varepsilon_A$ .

The control algorithm is given by

$$u = -k_A \widehat{s} - \widehat{\omega}_A^T \varphi_A(X_A), \quad k_A > 0. \quad (30)$$

**Theorem 2.** Consider the adaptive system (9) under Assumptions 1 and 2, controller (30) with HGO (21) and adaptive law (28) and (29). The filtered error  $s$  is semi globally uniformly ultimately bounded.

*Proof.* Let the Lyapunov function  $L_A = \frac{s^2}{2b_3}$ . Taking the time derivative of  $L_A$ , from eqs. (25)–(27) and (30) we get

$$\begin{aligned} \dot{L}_A &= \frac{s\dot{s}}{b_3} - \frac{\dot{b}_3 s^2}{2b_3^2} \\ &= \frac{s(a_3 + b_3 u + \widehat{v} - [0 \ \Lambda^T] \tilde{E})}{b_3} - \frac{\dot{b}_3 s^2}{2b_3^2} \\ &= s \frac{(-k_A b_3 \widehat{s} + b_3 (u_{\text{altitude}}^* - \widehat{\omega}_A^T \varphi_A) - [0 \ \Lambda^T] \tilde{Z})}{b_3} - \frac{\dot{b}_3 s^2}{2b_3^2} \\ &= s \left( -k_A s + k_A (s - \widehat{s}) + (\omega_A^{*T} \varphi_A + \zeta_A - \widehat{\omega}_A^T \varphi_A) - \frac{[0 \ \Lambda^T] \tilde{Z}}{b_3} \right) - \frac{\dot{b}_3 s^2}{2b_3^2} \\ &= -k_A s^2 + s \times \left( -k_A [\Lambda^T 1] \tilde{Z} + (u_{\text{altitude}}^* - \omega_A^{*T} \varphi_A) + \tilde{\omega}_A^T \varphi_A - \frac{[0 \ \Lambda^T] \tilde{Z}}{b_3} \right) - \frac{\dot{b}_3 s^2}{2b_3^2} \\ &= - \left( k_A + \frac{\dot{b}_3}{2b_3^2} \right) s^2 + s \times \left( -k_A [\Lambda^T 1] \tilde{Z} + (u_{\text{altitude}}^* - \omega_A^{*T} \varphi_A) + \tilde{\omega}_A^T \varphi_A - \frac{[0 \ \Lambda^T] \tilde{Z}}{b_3} \right) \\ &\leq - \left( k_A - \frac{b_{3d}}{2b_3^2} \right) s^2 + |s| \left( k_A c_{\lambda 1} \varepsilon h + \varepsilon_{Ae} + c_{\bar{w}} c_A + \frac{c_{\lambda 2} \varepsilon h}{b_3} \right), \end{aligned} \quad (31)$$

where  $c_{\bar{w}} = \varepsilon_A + |\omega^*|$ ,  $c_{\lambda 1} = \|[\Lambda^T \ 1]\|$ ,  $c_{\lambda 2} = \|[0 \ \Lambda^T]\|$  and  $\varepsilon_{Ae}$  is the supremum value of the  $u_{\text{altitude}}^*$  reconstruction error.

Let the constant  $c \triangleq \varepsilon_{Ae} + c_{\bar{w}} c_A + \frac{c_{\lambda 2} \varepsilon h}{b_3}$ . The last inequality of (30) is derived as

$$\dot{L}_A \leq - \left( k_A - \frac{b_{3d}}{2b_3^2} \right) s^2 + |s| (k_A c_{\lambda 1} \varepsilon h + c) = - \left( k_A - \frac{b_{3d}}{2b_3^2} \right) |s| \left( |s| - \frac{k_A c_{\lambda 1} \varepsilon h + c}{k_A - \frac{b_{3d}}{2b_3^2}} \right). \quad (32)$$

Take  $k_A > \frac{b_{3d}}{2b_3^2}$ . Then  $s$  is invariant to the set

$$\Omega_s = \left( |s| \leq \frac{k_A c_{\lambda 1} \varepsilon h + c}{k_A - \frac{b_{3d}}{2b_3^2}} \right). \quad (33)$$

By decreasing the observer parameter  $\varepsilon$  and increasing the input gain  $k_A$ , the radius can be made arbitrarily small.

**Remark 2.**

1. Based on HGO, the neural adaptive controller is quite simple without complex computation.
2. Only one RBFN with 4 inputs is employed to approximate the lumped uncertain nonlinear function (27).
3. There are no restrictive conditions on the design constants  $\varepsilon$ ,  $d_i$ 's,  $\lambda$ ,  $\gamma_A$ , and  $\varepsilon_A$ .

**4.2 Adaptive controller for velocity subsystem**

Define the velocity error as

$$Z_V = V - V_d. \tag{34}$$

The derivative of  $Z_V$  is as follows:

$$\dot{Z}_V = \dot{V} - \dot{V}_d = f_V + g_V \beta_c - \dot{V}_d = g_V [\beta_c + g_V^{-1} (f_V - \dot{V}_d)]. \tag{35}$$

According to Theorem 1, we denote

$$|g_V^{-1} (f_V - \dot{V}_d) - \omega_V^* \phi_V(X_V)| \leq \varepsilon_{Ve}, \tag{36}$$

where  $W_V^*$  is the optimal parameter vector for the approximation of  $g_V^{-1} (f_V - \dot{V}_d)$ .

The controller and the update law of  $\hat{W}$  are designed as follows:

$$\beta_c = -k_V Z_V - \rho_V \text{sgn}(Z_V) - \hat{\omega}_V \phi_V(X_V), \tag{37}$$

$$\dot{\hat{\omega}}_V = \tau Z_V \phi_V(X_V), \tag{38}$$

where  $X_V = [x_1, x_2, x_3, V, \dot{V}_d]^T$  and  $\rho_V > 0$ .  $\hat{\omega}_V$  is the estimation of  $\omega_V^*$  and  $\tau$  is the learning rate.

**Theorem 3.** Consider the adaptive system comprising (1) under Assumptions 1 and 2, controller (37) with adaptive law (38). The velocity of the HFV is globally uniformly bounded.

*Proof.* Let us consider the Lyapunov function candidate

$$L_V = \frac{Z_V^2}{2g_V} + \frac{1}{2} \tilde{\omega}_V^T \tau^{-1} \tilde{\omega}_V, \tag{39}$$

where  $\tilde{\omega}_V = \hat{\omega}_V - \omega_V^*$ .

The derivative of  $V$  is obtained as

$$\begin{aligned} \dot{L}_V &= -\frac{\dot{g}_V Z_V^2}{2g_V^2} + Z_V [-K_V Z_V - \rho_V \text{sgn}(Z_V) - \tilde{\omega}_V^T \phi_V(X_V) + \varepsilon_V] + \tilde{\omega}_V^T \tau^{-1} \dot{\tilde{\omega}}_V \\ &= -K_V Z_V^2 - \frac{\dot{g}_V Z_V^2}{2g_V^2} + Z_V [-\rho_V \text{sgn}(Z_V) + \varepsilon_V] + \tilde{\omega}_V^T (\tau^{-1} \dot{\tilde{\omega}}_V - Z_V \phi_V(X_V)) \\ &= -\left(K_V + \frac{\dot{g}_V}{2g_V^2}\right) Z_V^2 + Z_V [-\rho_V \text{sgn}(Z_V) + \varepsilon_V] + \tilde{\omega}_V^T (\tau^{-1} \dot{\tilde{\omega}}_V - Z_V \phi_V(X_V)), \end{aligned} \tag{40}$$

where  $\varepsilon_V$  is the NN approximation error.

Taking  $K_V \geq \frac{g_{V,d}}{2g_V^2}$  and  $\rho_V \geq \varepsilon_{Ve}$ , we can determine

$$K = K_V + \frac{\dot{g}_V}{2g_V^2} \geq 0, \tag{41}$$

$$\dot{L}_V = -K Z_V^2 - Z_V (\rho_V - \varepsilon_V \text{sgn}(Z_V)) \text{sgn}(Z_V) \leq -K Z_V^2 - |Z_V| (\rho_V - \varepsilon_{Ve}). \tag{42}$$

Then  $\dot{L}_V \leq 0$  and the velocity of HFV is globally uniformly bounded.

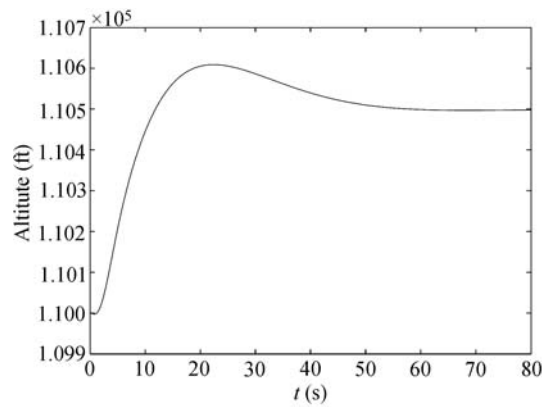


Figure 1 Altitude step response.

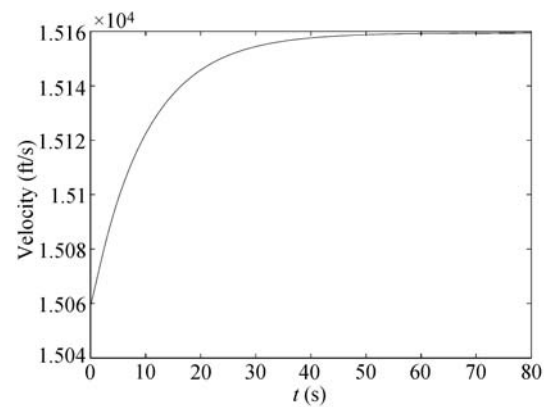


Figure 2 Velocity step response.

Here we employ saturation function to replace the sign function.

$$\text{sat}(Z_V) = \begin{cases} \frac{Z_V}{\zeta_V}, & \text{if } \left| \frac{Z_V}{\zeta_V} \right| < 1, \\ \text{sgn}(Z_V), & \text{otherwise.} \end{cases} \quad (43)$$

## 5 Numerical simulation

In this section, we verify the effectiveness and performance of the proposed adaptive neural controller. The control objective is to track the set-point set change ( $h_c$ ) with magnitude 500 ft while the airspeed steps from 15060 to 15160 ft/s. The following filters are used to generate the differentiable commands:

$$\frac{h_d}{h_c} = \frac{\omega_{n1}\omega_{n2}^2}{(s + \omega_{n1})(s^2 + 2\varepsilon_c\omega_{n2}s + \omega_{n2}^2)}, \quad (44)$$

$$\frac{V_d}{V_c} = \frac{\omega_{n3}}{s + \omega_{n3}}, \quad (45)$$

where  $\omega_{n1} = 5$ ,  $\omega_{n2} = 1$ ,  $\varepsilon_c = 0.7$ ,  $\omega_{n3} = 10$ .

The initial values of the weights of RBFs and the observer are set to zero. The parameters of altitude controller are given as:  $\varepsilon = 0.03$ ,  $\lambda = 2$ ,  $k_A = 15$ ,  $d_1 = 5$ ,  $d_2 = 3$ ,  $\varepsilon_A = 20$ ,  $\gamma_A = 10$ ,  $k_h = 0.15$ ,  $k_I = 0.01$ . The RBFs for the altitude controller are chosen to be  $3^4$  nodes of  $X_A$  with their centers being evenly spaced in  $[-0.4, 0.4; -0.3, 0.3; -3, 3; -0.3, 0.3]$ . The RBFs for the velocity controller are chosen to be  $3^5$  nodes of  $X_V$  with their centers being evenly spaced in  $[-0.4, 0.4; -0.3, 0.3; -3, 3; -0.3, 0.3; 15000, 15200]$ . The parameters of velocity controller are given as:  $k_V = 0.3$ ,  $\tau = 5 \times 10^{-4}$ ,  $\zeta_V = 1$ ,  $\rho_V = 0.2$ .

Figures 1 and 2 show the step tracking performance of altitude and velocity in spite of the system uncertainty. The control inputs of throttle setting and elevator deflection are given in Figures 3 and 4. Figure 5 shows that the real flight path angle is responding in quite the same way to the command derived from (7). In Figure 6, the estimation error of  $z_i$ 's is shown to be small enough which demonstrates the robustness of HGO. The neural output of  $u_{NN}$  is shown in Figure 7. Figure 8 shows the boundedness of  $|\hat{\omega}_A|$  and  $|\hat{\omega}_V|$ .

## 6 Conclusions

In this paper we have presented an adaptive controller for the longitudinal dynamics of hypersonic flight vehicle. The two subsystems are transformed into the output-feedback form. The altitude control is designed based on HGO with only one NN to approximate the lumped uncertain nonlinearity. The algorithm is much simpler than back-stepping scheme. It is proved that the filtered tracking error is guaranteed in the semiglobal sense and all the signals are uniformly bounded. For the velocity subsystem



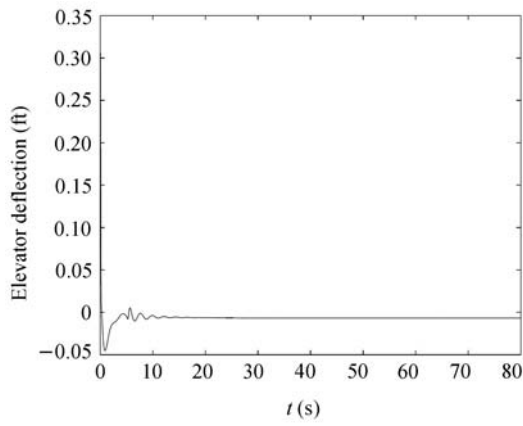


Figure 3 Elevator deflection.

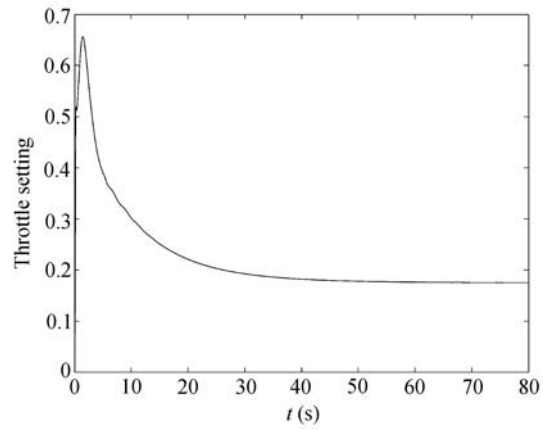


Figure 4 Throttle setting.

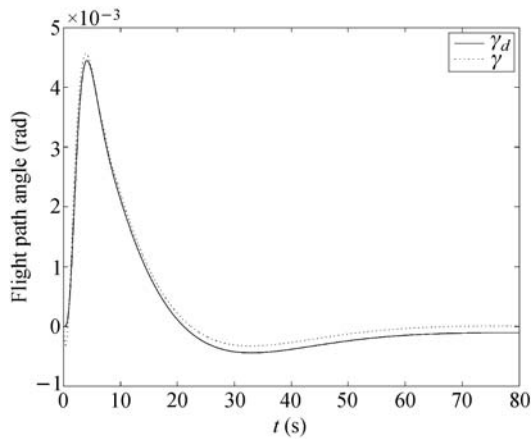


Figure 5 Simulation result  $\gamma_d, \gamma$ .

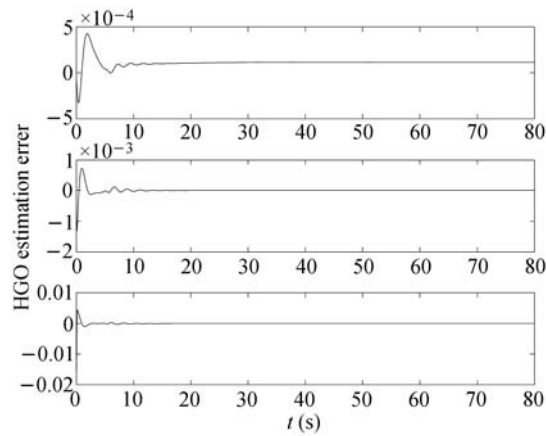


Figure 6 Simulation result  $\hat{E}$ .

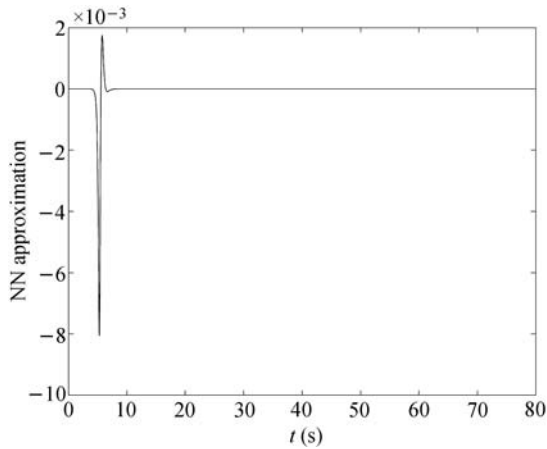


Figure 7 Neural output  $u_{NN}$ .

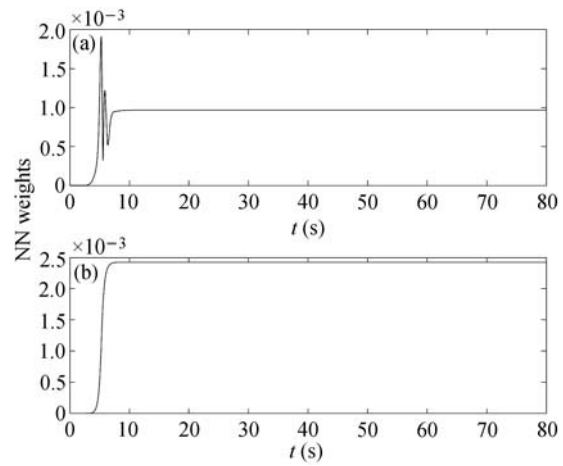


Figure 8 Trajectory of  $|\hat{\omega}_A|$  (a) and  $|\hat{\omega}_V|$  (b).

one adaptive NN controller is designed and the stability analysis is proved. Current work is mainly on the high fidelity of rigid-body model without flexibility effects which should be taken into consideration in the future.

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