SCIENCE CHINA
Information Sciences

• RESEARCH PAPERS •

June 2011 Vol. 54 No. 6: 1264–1273
doi: 10.1007/s11432-011-4214-y

Outage-optimal relaying through opportunistic hybrid forward cooperation

LI YuBo*, YIN QinYe & WANG JunSong

Ministry of Education Key Lab for Intelligent Networks and Network Security, Xi'an Jiaotong University, Xi'an 710049*, China*

Received May 13, 2010; accepted June 9, 2010; published online March 27, 2011

Abstract In this paper, a new scheme based on relay selection and data forwarding in multi-relay cooperative networks is introduced. Unlike most existing solutions focused on amplify-and-forward (AF) or decode-and-forward (DF) protocol separately, the proposed hybrid forward (HF) scheme takes advantage of both AF and DF protocols in relay forwarding. Based on theoretical analysis, a closed-form expression for the tight lower bound of its outage probability is derived, and further performance analysis is also presented. Then, in total power restricted networks, an optimal power allocation strategy between the source and the selected relay is developed to minimize the outage probability. Simulations are carried out finally, which validate the analysis and show that the HF scheme outperforms other existing ones in terms of outage probability.

Keywords cooperative diversity, opportunistic cooperation, outage probability, power allocation, diversity-multiplexing-tradeoff

Citation Li Y B, Yin Q Y, Wang J S. Outage-optimal relaying through opportunistic hybrid forward cooperation. Sci China Inf Sci, 2011, 54: 1264–1273, doi: 10.1007/s11432-011-4214-y

1 Introduction

Much attentions have been paid to cooperative diversity due to its ability to improve the performance of wireless networks in recent years [1]. Cooperations between the source and relays can provide independent paths to the destination, so as to improve the robustness against wireless multipath fading [2–13].

There are two basic cooperative protocols in cooperations: amplify-and-forward (AF) and decode-and-forward (DF) [6, 7]. In these years, various cooperation schemes based on AF and DF protocols have been proposed. After single-relay schemes being introduced and analyzed by Laneman et al. [2] and Nabar et al. [8], multi-relay schemes have been researched recently. One simple approach is using repetition transmissions among relay nodes, which provides diversity but meanwhile decreases the bandwidth efficiency. By taking advantage of distributed space-time coding (DSTC) among participating nodes, the DSTC scheme [9] greatly improves the spectral efficiency. However, strict synchronization is needed and code design is difficult due to the distributed trait in cooperative networks. Bletsas et al. [10] introduced the concept of "opportunistic cooperation", where only the best single relay is selected to forward the message. The opportunistic schemes achieve the same diversity-multiplexing-tradeoff (DMT) performance as the DSTC one but avoid code design and strict synchronization. In [11], based on AF and DF, both

*Corresponding author (email: heatonlyb@gmail.com)

opportunistic AF (Opp-AF) and opportunistic DF (Opp-DF) schemes were analyzed and considered to be outage-optimal.

Since either AF or DF protocol has its own merits and shortcomings, both Opp-AF and Opp-DF are not global optimal in terms of outage probability. One notable advantage of AF is its ability to forward information all times, even when the source-relay channel experiences outage. However, AF may cause error propagation by amplifying both signal and noise in forwarding, while DF only regenerates clean signals at the relay and performs better. To combine advantages of AF and DF, a scheme was proposed in [12], where soft decision and amplification are carried out at the single relay. Recently in [13], a hybrid decode-amplify-forward scheme in single-relay networks was proposed, where the relay utilizes AF or DF protocol alternatively based on whether it decodes correctly. By using maximum ratio combining (MRC) at the destination, this scheme was proved to outperform both AF and DF ones. However, these works are only focused on single-relay networks and cannot be applied in practice.

In this paper, originally from [10–13], an opportunistic hybrid forward (HF) scheme in multi-relay networks is proposed. In the HF scheme, the participating relays attempt to forward the message by DF protocol if they decode correctly and otherwise, adopt AF protocol. Then, the "best" relay is selected to join in the cooperation when it can provide the maximum instantaneous mutual information. Through theoretic analysis and numerical simulations, we indicate that, by taking advantage of AF, DF and opportunistic cooperation, the HF scheme outperforms both Opp-AF and Opp-DF in terms of outage probability. In addition, under the same power consumption, an optimal power allocation strategy between the source and the selected relay is developed to improve the outage performance further.

Notation: $|\cdot|$ denotes the cardinality of a set. $\Pr\{A\}$ is the probability of event A and $\binom{N}{n}$ represents the binomial coefficient, i.e. $\binom{N}{n} = \frac{N!}{(N-n)!n!}$. We introduce $\max\{\cdot\}$, $\min\{\cdot\}$ to signify the maximum and the minimum elements of a set, respectively. $\exp(\cdot)$ is the natural exponent of a number.

2 System model and protocols

We consider a dual-hop cooperative network consisting of a source node (s), a destination node (d) and M half-duplex relay nodes (m for $m \in \{1, \ldots, M\}$). The direct link exists between the source and the destination. We assume that instant error-free bidirectional channels exist between the destination and relays, hence control packets can be exchanged without delay and errors. For the purpose of theoretical analysis, it is further assumed that a perfect error-checking can be done at relays and the destination so that they can know exactly whether received message is correct. The cyclic redundancy check (CRC) can be used to implement this functionality.

The channel between any two nodes is described by a flat, quasi-static Rayleigh fading model. We model the channel coefficient between node i and node j as h_{ij} ($i \in \{s, 1, \ldots, M\}$ and $j \in \{1, \ldots, M, d\}$), which is a zero-mean, independent, circularly-symmetric complex Gaussian random variable with variance $1/\lambda_{ij}$. Furthermore, accomplished by RTS/CTS mechanism [10], perfect channel state information (CSI) is available at both the transmitter and the receiver based on hypothesis. The total transmitting power denoted by P_{total} is restricted, and the power allocated to the source and selected relay are

$$P_{\text{source}} = \xi P_{\text{total}}, \tag{1}$$

$$P_{\text{relay}} = (1-\xi) P_{\text{total}}, \tag{2}$$

where $\xi, (1-\xi) \in [0,1]$ indicate the proportions of P_{total} allocated to the source and the selected relay, respectively. Especially, $\xi = 0.5$ means no power allocation strategy adopted. The additive noise at each receiver is represented as a zero-mean, complex Gaussian random variable with the same variance N_0. For simplicity, by introducing overall SNR $\rho = P_{\text{total}}/N_0$, we signify the average received SNR from the source and the selected relay as $\rho_{\text{source}} = \xi\rho$ and $\rho_{\text{relay}} = (1-\xi)\rho$, respectively. Afterward, we introduce the notion $\gamma_{ij} = \rho_{ij}|h_{ij}|^2$ to represent the instantaneous SNR between node i and node j, where

$$\rho_{ij} = \begin{cases} \rho_{\text{source}}, & \text{if } i = s; \\ \rho_{\text{relay}}, & \text{if } i \neq s. \end{cases} \tag{3}$$

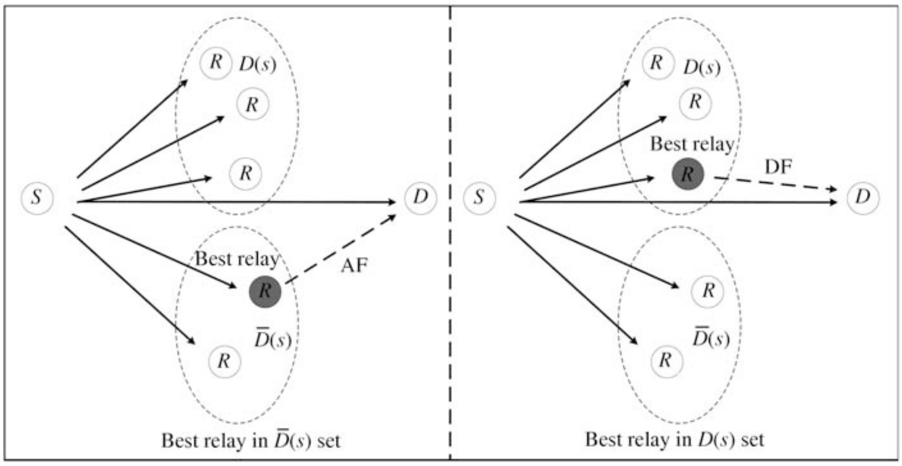

Figure 1 Proposed hybrid forward scheme.

Obviously, γ_{ij} obeys exponential distribution with parameter λ_{ij}/ρ_{ij}. Finally, the input codewords with rate R are assumed to be obtained from a random Gaussian codebook [14, 15].

Now we discuss the principles of the HF scheme in detail shown in Figure 1. Same as most existing schemes, every cooperation period is divided into two phases. Solid lines and dashed lines in Figure 1 stand for transmissions in the first and second phases, respectively. In the first phase, the source broadcasts a packet while the destination and participating relays keep listening. The relays which correctly decode the packet constitute the DF set $D(s)$ and others constitute AF set $\bar{D}(s)$, where $|D(s)|+|\bar{D}(s)|=M$. At the beginning of the second phase, the destination broadcasts a flag. As soon as any relay receives the flag, it starts a timer inversely proportional to γ'_{md} of the mth relay. The definition of γ'_{md} is described as follows:

$$\gamma'_{md}=\begin{cases}\gamma_{md}, & \text{if } m\in D(s),\\ f(\gamma_{sm},\gamma_{md}), & \text{if } m\in \bar{D}(s),\end{cases} \tag{4}$$

where $f(x,y)=xy/(1+x+y)$ mentioned in [2]. γ'_{md} characterizes the received SNR at the destination in the second phase by the mth relay forwarded. That is, the relay which can provide maximum instantaneous SNR may set up minimum timer. Then, obviously, the relay whose timer expires first will be selected to forward the packet received previously by either AF or DF protocol, according to the set it belongs to. And the other relays keep listening before their timers reduce to zero and will be back off when the vote is finished. In this manner, the cooperation can be carried out by a distributed fashion. Detailed timer mechanism is provided in [10] and the probability of packets collision is also analyzed therein, which are out of the scope of this paper.

3 Performance analysis

Theoretical analysis of the HF scheme is provided in this section. For simplicity, the quasi-symmetrical networks are applied for theoretic derivation, where channels between the source and relays have identical distributions, and this also holds for the relay-destination channels. Thus, λ_{sm} and λ_{md} for any $m\in\{1,2,\ldots,M\}$ can be represented as λ_{sr} and λ_{rd}, respectively. However, in fact, the HF scheme is still effective in asymmetrical networks, which will be validated in simulations hereinafter. We also introduce the concept of symmetrical networks where all channels have identical distributions, i.e. $\lambda_{sr}=\lambda_{rd}=\lambda_{sd}$ to distinguish from the concept of quasi-symmetrical networks.

3.1 Outage probability

We adopt MRC to combine received data in the two phases at the destination. When the kth relay is selected to forward data, the mutual information between the source and the destination can be expressed as

$$I_{\rm HF} = \frac{1}{2}\log_2(1+\gamma_{sd}+\gamma'_{kd}), \tag{5}$$

where γ'_{kd} is defined in (4) and 1/2 denotes two phases in each cooperation period. Outage events occur when $I_{\rm HF}$ falls below a specified target transmission rate R, i.e. $I_{\rm HF} < R$. Therefore, the outage probability is

$$\mathcal{P}_{\rm out} = \Pr\{I_{\rm HF} < R\} = \Pr\{\gamma_{sd}+\gamma'_{kd} < u\}, \tag{6}$$

where $u = 2^{2R}-1$ denotes the SNR threshold of outages. That is, the outage event will occur when the received SNR at the destination is below u.

Lemma. By introducing notations $\tilde{\lambda}_{sr}$, $\tilde{\lambda}_{rd}$ and $\tilde{\lambda}_{sd}$ to represent $\lambda_{sr}/\xi\rho$, $\lambda_{rd}/(1-\xi)\rho$, $\lambda_{sd}/\xi\rho$ respectively, the outage probability of the HF scheme is lower bounded by

$$\begin{aligned}\mathcal{P}_{\rm out}^{lb} = &\sum_{m=0}^{M}\sum_{n=0}^{M-m}\sum_{k=0}^{n}\sum_{l=0}^{m}\begin{pmatrix}M\\m\end{pmatrix}\begin{pmatrix}M-m\\n\end{pmatrix}\begin{pmatrix}n\\k\end{pmatrix}\begin{pmatrix}m\\l\end{pmatrix}\\ &\times(-1)^{k+l}\tilde{\lambda}_{sd}\exp(-\tilde{\lambda}_{sd}u)(\exp(-\tilde{\lambda}_{sr}u))^{m+n-k}\times(1-\exp(-\tilde{\lambda}_{sr}u))^{M-m-n}\times g(u),\end{aligned} \tag{7}$$

where

$$g(u) = \begin{cases} u, & \text{if } f(n,k,l)=0,\\ \dfrac{1-\exp(-f(n,k,l)u)}{f(n,k,l)}, & \text{if } f(n,k,l)\neq 0\end{cases} \tag{8}$$

and

$$f(n,k,l) = \tilde{\lambda}_{rd}n+\tilde{\lambda}_{sr}k+\tilde{\lambda}_{rd}l-\tilde{\lambda}_{sd}. \tag{9}$$

Proof. See Appendix [16, 17].

Although $\mathcal{P}_{\rm out}^{lb}$ is the lower bound of the outage probability $\mathcal{P}_{\rm out}$, it is tight enough to analyze the HF scheme especially in high SNR regime, which will be illustrated by simulations. Thus, we can approximate $\mathcal{P}_{\rm out}$ by $\mathcal{P}_{\rm out}^{lb}$ to evaluate the performance of the HF scheme hereinafter.

3.2 Diversity order and DMT performance

In this subsection, we will prove that the full diversity order $M+1$ is achieved for the HF scheme and then, evaluate its performance by DMT, which is introduced in [18].

Consider a family of codes C_ρ operating at SNR ρ with rates $R(\rho)$. If $\mathcal{P}(R)$ is the outage probability of the channel for rate R, then the multiplexing gain r and the diversity order d are defined as

$$r \triangleq \lim_{\rho\to\infty}\frac{R(\rho)}{\log_2\rho}, \qquad d \triangleq -\lim_{\rho\to\infty}\frac{\log_2\mathcal{P}(R)}{\log_2\rho}. \tag{10}$$

Based on the definition of diversity order, we have

Corollary 1. The HF scheme achieves full diversity order $M+1$, where M is the number of relays.

Proof: By introducing the approximation $1-\exp(-x)\doteq x$ when $x\to 0$, we can simplify (A3) when $\rho\to\infty$:

$$\begin{aligned}\Pr\{outage\,|\,|D(s)|=m\} &\doteq \int_0^u \frac{\lambda_{sd}}{\xi\rho}\left(\frac{\lambda_{rd}}{(1-\xi)\rho}y\right)^m\times\left(\frac{\frac{\lambda_{sr}}{\xi\rho}y+\frac{\lambda_{rd}}{(1-\xi)\rho}y-\frac{\lambda_{rd}}{(1-\xi)\rho}y}{\frac{\lambda_{sr}}{\xi\rho}u}\right)^{M-m}dy\\ &= \frac{c_1}{\rho^{m+1}u^{M-m}}\int_0^u y^M dy = c_2\rho^{-m-1}u^{m+1}.\end{aligned} \tag{11}$$

Then, substituting (11) and (A2) into (A1), we get

$$\mathcal{P}_{\rm out} \doteq \sum_{m=0}^{M}\begin{pmatrix}M\\m\end{pmatrix}\left(\frac{\lambda_{sr}}{\xi\rho}u\right)^{M-m}c_2\rho^{-m-1}u^{m+1} = c_3u^{(M+1)}\rho^{-(M+1)}, \tag{12}$$

where c_1, c_2 and c_3 are constants. From the definition in (10), we obtain $d = M + 1$ simply.

Afterward, we discuss DMT of the proposed scheme:

Corollary 2. The diversity-multiplexing-tradeoff of the HF scheme is

$$\Delta_{\mathrm{HF}}(r) = (M+1)(1-2r)^+, \tag{13}$$

where $(\cdot)^+ \triangleq \max\{\cdot, 0\}$.

Proof. In high SNR regime, from the first definition in (10), we assume that a code with rate $R = r\log_2\rho$ is selected. And then we get $u \doteq \rho^{2r}$. Eq. (12) can be rewritten as

$$\mathcal{P}_{\mathrm{out}} \doteq c_3\rho^{2r(M+1)}\rho^{-(M+1)} \doteq c_3\rho^{-(M+1)(1-2r)^+}. \tag{14}$$

Thus, based on the definition of DMT [18], we have $\Delta_{HF}(r) = (M+1)(1-2r)^+$.

Corollaries 1 and 2 demonstrate that the HF scheme achieves the same diversity order and DMT performance as Opp-AF and Opp-DF ones analyzed in [10]. However, the HF scheme outperforms these alternatives in terms of outage probability, which will be shown in simulations.

3.3 Optimal power allocation

When total power P_{total} in one cooperation period is restricted, the power allocation between the source and the selected relay may greatly affect the outage probability of the networks. The parameter ξ denotes the proportion of power allocating to the source. Intuitively, a tradeoff exists for power allocation between the source and the selected relay. On the one hand, too big a ξ in networks leads to power deficiency at the selected relay. In this case, the destination can hardly decode relay's packet correctly and the diversity gain is lost. On the other hand, small ξ may cause outages with high probability in the source-relay channels and source-destination channel, so that the communication may be interrupted. Therefore, it is desirable to give the principle of choosing the optimal power allocation parameter ξ_{opt} according to the outage formulation derived before. To minimize the outage probability, ξ_{opt} is selected to correspond to

$$\xi_{\mathrm{opt}} = \arg\min_{0\leqslant\xi\leqslant1} \mathcal{P}_{\mathrm{out}}(\xi). \tag{15}$$

Due to the complicated expression of the outage probability, it is really difficult to obtain ξ_{opt} with a closed-form. However, ξ_{opt} can be acquired from one-dimension exhaustive search, whose accuracy depends on the search step. Figure 2(a) illustrates the outage probability as a function of ξ with different SNR when $M = 2$. ξ_{opt} are denoted by asterisks. Generally, ξ_{opt} is almost unchanged at high SNR($\geqslant$ 16 dB) and we can consider it as a constant. Therefore, in high SNR regime, ξ_{opt} can be fixed prior to transmission without any real time updating. In Figure 2(b), the outage probability versus different M with SNR=20 dB is presented. It shows that more power is allocated to the selected relay when M increases, because the best relay-destination channel will be more robust when more relays participate in relay selections. Another observation from Figure 2 is that the bottoms of the curves are flat, which implies that the small inaccuracy of ξ_{opt} will not affect the outage probability greatly. This makes it sufficient to use a relatively larger step in the searching algorithm in order to reduce the computing complexity.

4 Simulations and discussions

Simulations for the outage probability of the proposed HF scheme and existing ones are represented in this section. R=1 bps/Hz is applied for end-to-end spectral efficiency. ρ is denoted by SNR directly in simulations.

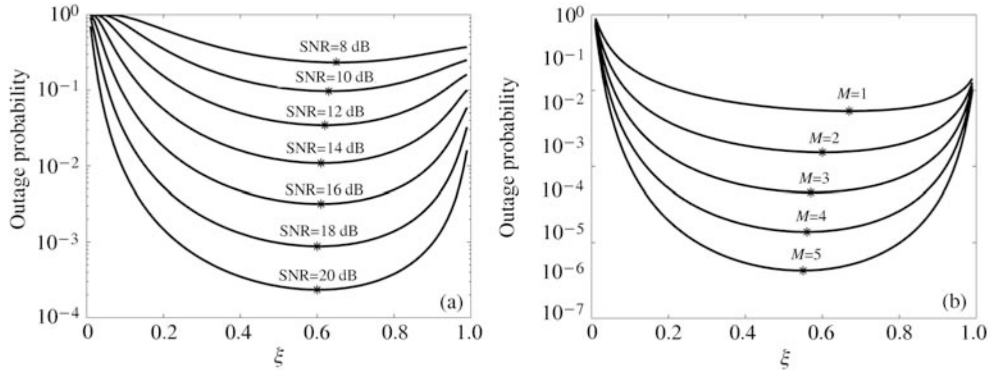

Figure 2 Outage probability influenced by power allocation parameter ξ. (a) Power allocation with different SNR; (b) power allocation with different M.

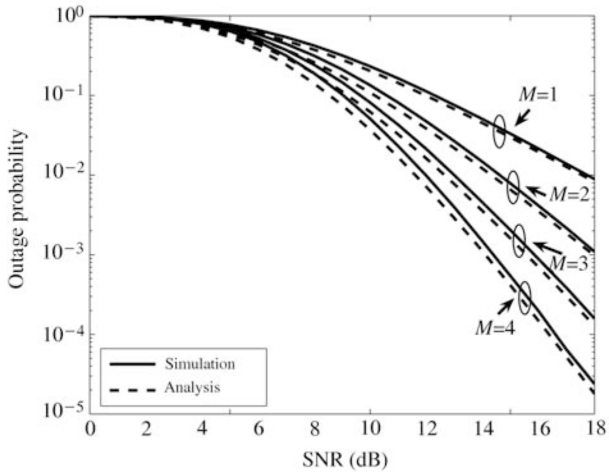

Figure 3 Theoretical lower bound performance.

4.1 Theoretical lower bound

Firstly, simulations are carried out to evaluate the theoretical lower bound (7) derived. In Figure 3, the outage probability of the HF scheme versus different numbers of relays is illustrated. In simulations, we use symmetrical networks with $\lambda_{sr} = \lambda_{rd} = \lambda_{sd} = 1$ and no power allocation strategy is adopted ($\xi = 0.5$). It shows that the derived lower bound of the outage probability matches well to the simulation result especially in high SNR regime, validating the correctness of aforementioned theoretical analysis. In particular, the HF scheme degenerates to the hybrid decode-amplify-forward scheme [13] when $M = 1$.

4.2 Performance comparisons and power allocation discussions

The outage probability of the HF scheme compared with Opp-AF and Opp-DF ones is illustrated in Figure 4, and the performance comparison of the HF scheme between no power allocation and optimal power allocation is also presented therein. No power allocation strategy is adopted by both Opp-AF and Opp-DF schemes and then, quasi-symmetrical networks with $\lambda_{sr} = 2$, $\lambda_{rd} = 1$, $\lambda_{sd} = 1.5$ in Figure 4(a) and symmetrical networks with $\lambda_{sr} = \lambda_{rd} = \lambda_{sd} = 1$ in Figure 4(b) are applied in simulations. It shows that the HF scheme outperforms the two comparative ones in terms of outage probability with no power allocation strategy, which shows the superiority of the proposed scheme. Then, with optimal power allocation, the outage performance can be remarkably improved in quasi-symmetrical networks, while nearly no benefit is obtained with power allocation strategy in symmetrical networks.

Table 1 shows additional SNR gains benefited from the optimal power allocation in various network topologies when the outage probability reaches 10^{-4} and SNR at 20 dB, the corresponding ξ_{opt} being

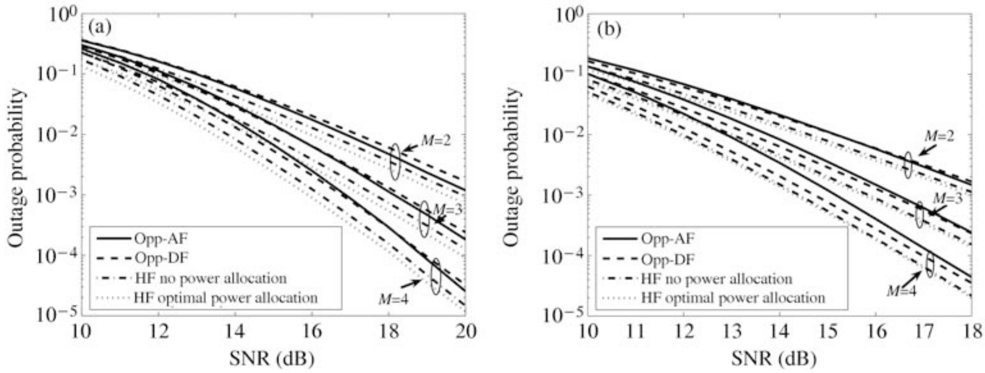

Figure 4 Outage probability comparisons with different schemes. (a) In quasi-symmetrical networks; (b) in symmetrical networks.

Table 1 Additional SNR gains employed by optimal power allocation

Scenarios			Gains (dB)		
λ_{sr}	λ_{rd}	λ_{sd}	$M=1$	$M=3$	$M=5$
1	1	1	0.27(0.67)	0.08(0.57)	0.04(0.55)
2	1	1	0.67(0.72)	0.33(0.65)	0.26(0.63)
2	1	2	0.67(0.72)	0.33(0.65)	0.26(0.63)
6	1	1	1.26(0.81)	0.95(0.75)	0.87(0.74)
4	2	3	0.67(0.73)	0.32(0.65)	0.25(0.63)
1	2	1	0.18(0.63)	0.00(0.50)	0.01(0.47)
1	6	1	0.03(0.59)	0.11(0.41)	0.22(0.37)

in the bracket. On the one hand, remarkable gain is obtained in the network with unbalanced topology, and this gain is mostly influenced by the ratio $\lambda_{sr}/\lambda_{rd}$ but almost independent of λ_{sd}. It is notable that more power is allocated to the selected relay when the channel condition between the selected relay and the destination becomes worse. Generally, since the source-relay-destination link is more reliable than the source-destination one based on opportunistic relay selection, the power is preferred to be allocated to guarantee the performance of the source-relay-destination link. Therefore, it is wise to allocate more power to the selected relay when the condition of relay-destination channel becomes worse, in the case where the additional benefit obtained from relay forwarding surpasses the deficiency caused by reducing the transmitting power at the source. On the other hand, in symmetrical networks, we only get neglectable additional gains from the optimal power allocation especially when M is large. That means, in practice, we should not utilize any power allocation strategy in order to reduce system complexity in symmetrical networks.

4.3 Outage probability in asymmetrical networks

Now we evaluate the outage performance of the HF scheme in asymmetrical networks. Consider an asymmetrical network model subsequently: The nodes of the whole network are distributed in a square area. The source and the destination are located at diagonal corners and relays are generated randomly in the square. We assume that the variance of the channel coefficient $1/\lambda_{ij}$ is determined by the distance d_{ij} between node i and node j, i.e. $1/\lambda_{ij} = d_{ij}^{-\eta}$, where $\eta = 2$ denotes the path loss exponent. Without loss of generality, we assume $\lambda_{sd} = 1$, which means distances are normalized by d_{sd}. So we have $\lambda_{ij} = (d_{ij}/d_{sd})^2$. At last, no power allocation strategy is applied. The outage probability of such a network is shown in Figure 5 with different M, compared with the Opp-AF and Opp-DF schemes. Explicitly, the HF scheme outperforms others and full diversity order is also asymptotically achieved.

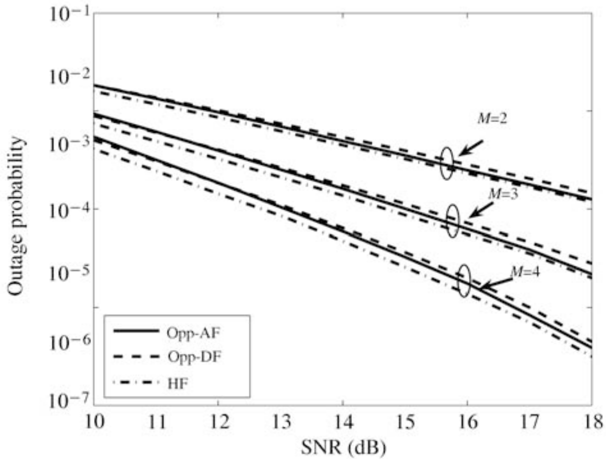

Figure 5 Outage probability comparisons in asymmetrical networks.

5 Conclusions

In this paper, an opportunistic HF scheme is proposed in multi-relay networks which benefits from both AF and DF protocols. The outage probability is analyzed, and a closed-form expression for its lower bound is derived. Analysis shows that the HF scheme achieves full diversity order of $M+1$, and has the same DMT performance $(M+1)(1-2r)^+$ as both AF and DF based opportunistic schemes. Afterward, the optimal power allocation strategy is developed to improve the outage performance in power restricted networks. It shows that the optimal proportion of power allocation is nearly unchanged in high SNR region, and can be fixed prior to the transmission. Then, the optimal power allocation strategy mainly aims at the enhancement of relay link due to its reliability provided by opportunistic relay selection. We conclude that the power allocation is urgent in unbalanced networks but may not be used in symmetrical networks in order to reduce system complexity. Simulations are carried out finally. It validates that the derived lower bound of outage probability is tight especially in high SNR regime, and also shows that, by combining the merits of AF, DF and opportunistic cooperation, the HF scheme outperforms both Opp-AF and Opp-DF schemes in terms of outage probability.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant Nos. 60772095, 60971113), the Ph.D. Programs Foundation of Ministry of Education of China (Grant No. 200806980020), and the Foundation for Innovative Research Groups of the NSFC (Grant No. 60921003).

References

1 Sendonaris A, Erkip E, Aazhang B. User cooperation diversity—part I: system description. IEEE Trans Commun, 2003, 51: 1927–1938

2 Laneman J N, Tse D N C, Wornell G W. Cooperative diversity in wireless networks: efficient protocols and outage behavior. IEEE Trans Inf Theory, 2004, 50: 3062–3080

3 Yin Q Y, Zhang Y, Ding L, et al. Cooperation diversity: a new spatial diversity technique (in Chinese). J Xi'an Jiaotong Univ, 2005, 39: 551–557

4 Wang J, Milstein L B. CDMA overlay situations for microcellular mobile communications. IEEE Trans Commun, 1995, 43: 603–614

5 Wang J, Chen J. Performance of wideband CDMA systems with complex spreading and imperfect channel estimation. IEEE J Select Areas Commun, 2001, 19: 152–163

6 Kramer G, Gastpar M, Gupta P. Cooperative strategies and capacity theorems for relay networks. IEEE Trans Inf Theory, 2005, 51: 3037–3063

7 Gastpar M, Vetterli M. On the capacity of large Gaussian relay networks. IEEE Trans Inf Theory, 2005, 51: 765–779

8 Nabar R U, Bölcskei H, Kneubühler F W. Fading relay channels: performance limits and space-time signal design. IEEE J Select Areas Commun, 2004, 22: 1099–1109
9 Laneman J N, Wornell G W. Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks. IEEE Trans Inf Theory, 2003, 49: 2415–2425
10 Bletsas A, Khisti A, Reed D P, et al. A simple cooperative diversity method based on network path selection. IEEE J Select Areas Commun, 2006, 24: 659–672
11 Bletsas A, Shin H, Win M Z. Cooperative communications with outage-optimal opportunistic relaying. IEEE Trans Wirel Commun, 2007, 6: 3450–3460
12 Bao X, Li J. Efficient message relaying for wireless user cooperation: decode-amplify-forward (DAF) and hybrid DAF and coded-cooperation. IEEE Trans Wirel Commun, 2007, 6: 3975–3984
13 Duong T Q, Zepernick H J. On the performance gain of hybrid decode-amplify-forward cooperative communications. EURASIP J Wirel Commun Netw, 2009, Article ID 479463
14 Bletsas A, Khisti A, Win M Z. Opportunistic cooperative diversity with feedback and cheap radios. IEEE Trans Wirel Commun, 2008, 7: 1823–1827
15 Tannious R, Nosratinia A. Spectrally-efficient relay selection with limited feedback. IEEE J Select Areas Commun, 2008, 26: 1419–1428
16 Barua B, Ngo H Q, Shin H. On the sep of cooperative diversity with opportunistic relaying. IEEE Commun Lett, 2008, 12: 727–729
17 Abramowitz M, Stegun I A. Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. 9th ed. New York: Dover Publications, 1970
18 Zheng L, Tse D N C. Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels. IEEE Trans Inf Theory, 2003, 49: 1073–1096

Appendix: Proof of Lemma

Using the law of total probability, we get

$$\mathcal{P}_{\text{out}} = \sum_{m=0}^{M} \Pr\{|D(s)| = m\} \Pr\{outage \mid |D(s)| = m\}, \tag{A1}$$

where $\Pr\{|D(s)| = m\}$ signifies the probability of m relays in $D(s)$ set. Since different channels suffer independent fading and γ_{ij} obeys exponential distribution, we have

$$\Pr\{|D(s)| = m\} = \begin{pmatrix} M \\ m \end{pmatrix} \left(\exp\left(-\frac{\lambda_{sr}}{\xi\rho}u\right)\right)^m \left(1 - \exp\left(-\frac{\lambda_{sr}}{\xi\rho}u\right)\right)^{M-m}. \tag{A2}$$

The second term in the sum of (A1) denotes the outage probability under the condition of m relays in $D(s)$ set. Derived from (6), it can be rewritten as

$$\Pr\{outage \mid |D(s)| = m\} = \Pr\{\gamma_{sd} + \gamma'_{kd} < u \mid |D(s)| = m\} = \int_0^u F_{\gamma'_{kd},m}(y) f_{\gamma_{sd},m}(u-y)dy, \tag{A3}$$

where $F_{\gamma'_{kd},m}(y)$ and $f_{\gamma_{sd},m}(y)$ are used to describe the cumulative distribution function (CDF) of γ'_{kd} and the probability density function (PDF) of γ_{sd} under the condition of $|D(s)| = m$, respectively. γ_{sd} obeys exponential distribution with parameter $\lambda_{sd}/\xi\rho$ and is independent of m; therefore,

$$f_{\gamma_{sd},m}(y) = \frac{\lambda_{sd}}{\xi\rho} \exp\left(-\frac{\lambda_{sd}}{\xi\rho}y\right). \tag{A4}$$

Mathematically, the kth relay is selected when γ'_{kd} corresponds to

$$\gamma'_{kd} = \max_{k\in\{k_1,k_2\}} \left\{ \max_{k_1\in D(s)} \{\gamma_{k_1 d}\}, \max_{k_2\in \bar{D}(s)} \{f(\gamma_{sk_2}, \gamma_{k_2 d})\} \right\}. \tag{A5}$$

Then we have

$$F_{\gamma'_{kd},m}(y) = \left(1 - \exp\left(-\frac{\lambda_{rd}}{(1-\xi)\rho}y\right)\right)^m \times \prod_{k\in\{1,\ldots,M-m\}} \Pr\{f(\gamma_{sk}, \gamma_{kd}) < y \mid \gamma_{sk} < u\}, \tag{A6}$$

where the condition $\gamma_{sk} < u$ indicates relays decoding error in the first phase. For simplicity, we denote $\bar{\gamma}_k = f(\gamma_{sk}, \gamma_{kd})$. $\bar{\gamma}_k$ is a MacDonald random variable [16] whose CDF is

$$F_{\bar{\gamma}_k}(x) = 1 - 2\exp\bigg(-\bigg(\frac{\lambda_{sr}}{\xi\rho} + \frac{\lambda_{rd}}{(1-\xi)\rho}\bigg)x\bigg) \times \sqrt{\frac{\lambda_{sr}\lambda_{rd}}{\xi(1-\xi)\rho^2}x(x+1)} \times K_1\bigg(2\sqrt{\frac{\lambda_{sr}\lambda_{rd}}{\xi(1-\xi)\rho^2}x(x+1)}\bigg), \quad \text{(A7)}$$

where $x \geqslant 0$ and $K_v(x)$ is the vth order modified Bessel function of the second kind. When ρ is sufficiently large, $K_1(x)$ is tightly upper bounded by $1/x$ [17, (9.6.9)]. Thus, in high SNR regime, $F_{\bar{\gamma}_k}(x)$ becomes

$$F_{\bar{\gamma}_k}(x) \doteq 1 - \exp\bigg(-\bigg(\frac{\lambda_{sr}}{\xi\rho} + \frac{\lambda_{rd}}{(1-\xi)\rho}\bigg)x\bigg). \quad \text{(A8)}$$

From the equation above, we find that $\bar{\gamma}_k$ can be tightly upper bounded by $\min(\gamma_{sk}, \gamma_{kd})$. It is obvious that $\Pr\{\bar{\gamma}_k < y \mid \gamma_{sk} < u\}$ corresponds to 1 under the condition of $y > u$, and when $y < u$,

$$\begin{aligned}\Pr\{\bar{\gamma}_k < y \mid \gamma_{sk} < u\} &\doteq \Pr\{\min(\gamma_{sk}, \gamma_{kd}) < y \mid \gamma_{sk} < u\} \\ &= 1 - \Pr\{\gamma_{sk} > y, \gamma_{kd} > y \mid \gamma_{sk} < u\} \\ &= 1 - \frac{\Pr\{y < \gamma_{sk} < u\}\Pr\{\gamma_{kd} > y\}}{\Pr\{\gamma_{sk} < u\}} \\ &= 1 - \frac{(\exp(-\frac{\lambda_{sr}}{\xi\rho}y) - \exp(-\frac{\lambda_{sr}}{\xi\rho}u))\exp(-\frac{\lambda_{rd}}{(1-\xi)\rho}y)}{1 - \exp(-\frac{\lambda_{sr}}{\xi\rho})u}. \end{aligned} \quad \text{(A9)}$$

Then, we denote $g_1(y) = \Pr\{\bar{\gamma}_k < y \mid \gamma_{sk} < u\}$ to facilitate the expression, and also replace $\lambda_{sr}/\xi\rho$, $\lambda_{rd}/(1-\xi)\rho$, $\lambda_{sd}/\xi\rho$ with $\tilde{\lambda}_{sr}$, $\tilde{\lambda}_{rd}$ and $\tilde{\lambda}_{sd}$, respectively. We rewrite $g_1(y)$ as

$$g_1(y) \doteq \frac{1}{1-\exp(-\tilde{\lambda}_{sr}u)}(1 - \exp(-\tilde{\lambda}_{sr}u)) + \frac{1}{1-\exp(-\tilde{\lambda}_{sr}u)}\exp(-\tilde{\lambda}_{rd}y)(\exp(-\tilde{\lambda}_{sr}u) - \exp(-\tilde{\lambda}_{sr}y)), \quad \text{(A10)}$$

and then, substituting the result into (A6):

$$F_{\gamma'_{kd},m}(y) = (1 - \exp(-\tilde{\lambda}_{sr}y))^m g_1(y)^{M-m}. \quad \text{(A11)}$$

Afterward, by utilizing the binomial theorem

$$(a+b)^n = \sum_{k=0}^{n}\begin{pmatrix} n \\ k \end{pmatrix} a^k b^{n-k}, \quad \text{(A12)}$$

we can acquire the expansion of $g_1(y)^{M-m}$ and after some manipulations,

$$\begin{aligned} g_1(y)^{M-m} \doteq &\sum_{n=0}^{M-m}\sum_{k=0}^{n}\begin{pmatrix} M-m \\ n \end{pmatrix}\begin{pmatrix} n \\ k \end{pmatrix}(-1)^k \exp(-\tilde{\lambda}_{sr}u(n-k))(1-\exp(-\tilde{\lambda}_{sr}u))^{-n} \\ &\times \exp(-\tilde{\lambda}_{rd}ny - \tilde{\lambda}_{sr}ky). \end{aligned} \quad \text{(A13)}$$

Similarly, expanding the first term of (A11) by binomial theorem again, and then substituting the result into (A3), we get the outage probability under the condition of $|D(s)| = m$:

$$\begin{aligned} \Pr\{outage \mid |D(s)| = m\} \doteq &\sum_{n=0}^{M-m}\sum_{k=0}^{n}\sum_{l=0}^{m}\begin{pmatrix} M-m \\ n \end{pmatrix}\begin{pmatrix} n \\ k \end{pmatrix}\begin{pmatrix} m \\ l \end{pmatrix}(-1)^{k+l}\tilde{\lambda}_{sd} \\ &\times \exp(-\tilde{\lambda}_{sr}u(n-k) - \tilde{\lambda}_{sd}u)(1-\exp(-\tilde{\lambda}_{sr}u))^{-n}g(u), \end{aligned} \quad \text{(A14)}$$

where

$$\begin{aligned} g(u) &= \int_0^u \exp(-(\tilde{\lambda}_{rd}n + \tilde{\lambda}_{sr}k + \tilde{\lambda}_{rd}l - \tilde{\lambda}_{sd})y)dy \\ &= \int_0^u \exp(-f(n,k,l)y)dy \\ &= \begin{cases} u, & \text{if } f(n,k,l) = 0, \\ \dfrac{1-\exp(-f(n,k,l)u)}{f(n,k,l)}, & \text{if } f(n,k,l) \neq 0. \end{cases} \end{aligned} \quad \text{(A15)}$$

Finally, we insert (A2) and (A14) into (A1) and the final expression (7) in the lemma can be derived. Since upper bound is used to approximate $\bar{\gamma}_k$, the result we derived is the lower bound of its outage probability.