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Integral Barrier Lyapunov function-based adaptive control for switched nonlinear systems

Lei LIU[1], Yan-Jun LIU[1*], Aiqing CHEN[1], Shaocheng TONG[1] & C. L. Philip CHEN[2,3]

[1]*College of Science, Liaoning University of Technology, Jinzhou* 121001*, China;*
[2]*Computer Science and Engineering, South China University of Technology, Guangzhou* 510641*, China;*
[3]*Navigation College, Dalian Maritime University, Dalian* 116026*, China*

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Abstract This paper presents an adaptive control method for a class of uncertain strict-feedback switched nonlinear systems. First, we consider the constraint characteristics in the switched nonlinear systems to ensure that all states in switched systems do not violate the constraint ranges. Second, we design the controller based on the backstepping technique, while integral Barrier Lyapunov functions (iBLFs) are adopted to solve the full state constraint problems in each step in order to realize the direct constraints on state variables. Furthermore, we introduce the Lyapunov stability theory to demonstrate that the adaptive controller achieves the desired control goals. Finally, we perform a numerical simulation, which further verifies the significance and feasibility of the presented control scheme.

Keywords adaptive control, switched nonlinear systems, integral Barrier Lyapunov functions, backstepping technique, full state constraints

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1 Introduction

Engineering research and development is spurred by the dynamic growth of information and science, though many control systems in engineering fields become more complicated and often encounter the comprehensive characteristics of strong nonlinear, uncertainty, and time-varying parameters. These characteristics make it difficult to describe the control systems using specific mathematical models. So, the conventional control theory based on the precise mathematical model is hard to deal with the control problem of such complex systems. However, neural networks (NNs) and fuzzy logic systems (FLSs) can identify and learn unknown nonlinear dynamics [1, 2]; thus, the control method based on NNs or FLSs provides an effective method to resolve the modeling issue present in uncertain systems. Moreover, the method of combining the above approximation with adaptive controls as demonstrated in [3–6] for uncertain system is proposed in [7–15], especially in the field of constraint control.

Recently, the research direction focuses on the constraint control applications owing to its influence on the environment, safety, and other factors. Some variables in the actual system cannot be arbitrarily valued, but are restricted. If these variables do not meet the required limits, it may lead to performance degradation, or even result in system failure. Hence, the study of the constraint control problem has great practical significance. According to different constrained variables, constraint control problems can be divided into output constraint control problem [16–18], input constraint control problem [19, 20]

* Corresponding author (email: liuyanjun@live.com)

and state constraint control problem [21–25], while the state constraint problem includes partial state constraint problem and full state constraint problem. The output constraint can also be regarded as a partial state constraint in many cases. The authors in [26] proposed an adaptive distributed control law using the Barrier Lyapunov functions (BLFs) and a new speed function to ensure that every error vector converges to a predefined compact set in a finite time. Presently, the main method to overcome the state constraint issues is the BLF-based adaptive control method. In addition to the logarithmic BLF used for the above constraint problems, there is a common tangent BLF [27, 28], which can handle both constrained and unconstrained systems. Regardless if the method is based on logarithmic BLF or tangent BLF, there are conservative constraints that change state constraints into error constraints for realizing constraint control. However, the control strategy exploiting integral Barrier Lyapunov functions (iBLFs) can as well solve this problem (i.e., it can realize the direct constraint on the system states). Currently, a few numbers of research have been proposed using the constraint control method. Based on iBLFs, similar control schemes are proposed for nonlinear systems with state constraints and output constraints [29–31], so that the state variables of the system can be constrained within the specified limits. In particular, for the flexible crane system with a practical background, an iBLF-based adaptive control method is adopted in [30] to suppress vibration and solve the problem of uncertainty. In addition to the above constant constraints, time-varying constraints are frequently studied in [32], and the main research results are more in line with the requirements of the actual system. For example, the authors in [33, 34] applied time-varying constraint control to robot and vehicle active suspension systems, respectively. They combined their approach with NNs to deal with unknown terms, and finally realized the safety control of the actual system. Although these results are all obtained for non-switched systems, the results for switched systems using iBLFs seem to have not been reported yet. Motivated by the above discussion, hence, this paper studies the full state constraint control issue for the switched nonlinear system based on iBLFs.

The switched system, as a kind of special hybrid system, consists of a family of subsystems (continuous-time or discrete-time) and a switching rule, while the switching rule is used to guide how to switch between subsystems. The existence of the switching rule makes the switched system have some special properties. For example, the switched system often does not inherit all the characteristics of subsystems. Specifically, even if all subsystems are stable, the switched system may not be stable. Contrarily, even if all subsystems are unstable, the switched system may be stable by selecting the appropriate switching rule. Therefore, the switching rule plays a very significant role in the control of the switched system. Generally, the main classes of switching rules include arbitrary switching rule, average dwell time switching rule, and state-dependent maximum/minimum switching rule. Moreover, the adaptive intelligence scheme for the nonlinear switched systems using FLSs and NNs has been studied substantially in [35–38]. Futhermore, a few scholars have begun to focus their attention on the study of adding constraints to the switched systems [39–43]. For example, based on logarithmic BLF, the authors in [40, 41] proposed adaptive NN control schemes for random systems with output constraints. Then, fuzzy adaptive control approaches were obtained for nonlinear stochastic switching pure-feedback systems [42] with output constraints and non-strict-feedback stochastic switched nonlinear systems [43] with state constraints. However, there is no work on adaptive fuzzy control of the switched systems with full state constraints based on iBLFs.

Motivated by the above analysis, this paper studies the full state constraint control issue for a class of switched nonlinear systems with uncertainties. Furthermore, we consider the case that all subsystems are stable, and provide the sufficient conditions for the stability of the switched systems under arbitrary switching signals. Compared with the existing results, the main contributions are listed as follows.

(1) From the perspective of BLFs, we propose an adaptive control method based on iBLFs for the nonlinear systems and provide the necessary conditions that guarantee the stability of the resulted systems. In contrast to the previous results found in [16, 17, 21, 24, 27], in which log-type or tan-type BLFs are developed, we introduce the integral BLFs. In this case, we realize the direct constraints rather than the indirect constraints proposed in [16, 17, 21, 24, 27]. To the best of our knowledge, there are few relevant results for nonlinear systems using iBLF framework.

(2) From the perspective of the controlled plants, we consider the switched nonlinear systems. Al-

though the constraint problems are addressed in lots of existing studies [18, 22, 28, 30, 34], only the single mode is investigated in those studies. Although this paper concentrates on the switched systems (multiple mode) for constraint problems, we introduce the iBLFs to constrain all state variables in the switched systems. Also, the derivative problem of the variable limit integral is solved using an integral mean value theorem, which makes the switched systems more valuable.

2 System descriptions

The following switched nonlinear systems are considered:

$$\begin{cases} \dot{x}_i = f_{i,\sigma(t)}\left(\bar{x}_i\right) + g_i\left(\bar{x}_i\right) x_{i+1},\ i = 1, \ldots, n-1, \\ \dot{x}_n = f_{n,\sigma(t)}\left(\bar{x}_n\right) + g_n\left(\bar{x}_n\right) u, \\ y = x_1, \end{cases} \tag{1}$$

where $x = [x_1, \ldots, x_n]^{\mathrm{T}}$ is the system state, y is the output of systems, $\sigma(t)$: $[0, +\infty) \to \Gamma = \{1, 2, \ldots, \tau\}$ is the switching signal, $u_k \in \mathbb{R}$ is the control input of the k-th subsystem. Here, for any $i = 1, 2, \ldots, n$ and $k = 1, 2, \ldots, \tau$, $f_{i,k}\left(\bar{x}_i\right)$ and $g_i\left(\bar{x}_i\right)$ are smooth unknown system functions.

The control objective of this paper is to design adaptive intelligent controllers such that

(1) All signals in the resulted system are semi globally uniformly; and ultimately bounded (SGUUB);

(2) The system output tracks the given signal y_d as soon as possible;

(3) The full state constraint conditions $|x_i(t)| < |k_{c_i}|$ are satisfied.

In order to realize the follow-up controller design, the introduction of assumptions and lemmas are crucial.

Assumption 1 (Such as [31]). For $k_{c_1} : \mathbb{R}_+ \to \mathbb{R}$, there exist a function $Y_0(t) : \mathbb{R}_+ \to \mathbb{R}_+$, and constants $Y_i > 0$, $i = 1, 2, \ldots, n$, so that the desired trajectory y_d and its time derivatives satisfy

$$|y_d(t)| \leqslant Y_0(t) < k_{c_1}, \quad \left|y_d^{(i)}(t)\right| < Y_i, \quad \forall t \geqslant 0.$$

Assumption 2 (The same as [44]). Let $\hat{\Theta}_{i,j}$ denote the estimation of the $\Theta_{i,j}$. This paper assumes that $\hat{\Theta}_{i,j}(0)$, $j = 1, 2, \ldots, n_i$ are required to satisfy $\hat{\Theta}_{i,j}(0) \geqslant 0$ such that $\hat{\Theta}_{i,j} \geqslant 0$.

Assumption 3. There exit positive constants $\underline{g}_i$ and $\bar{g}_i$, satisfying

$$0 < \underline{g}_i \leqslant g_i\left(\bar{x}_i\right) < \infty, \quad i = 1, 2, \ldots, n.$$

Remark 1. In reality, a lot of adaptive control researches are based on Assumption 3, such as [44, 45]. Similarly, for generality, this study assumes that $g_i(\bar{x}_i) > 0$. We structure the controller and virtual controller by utilizing the bounds of $g_i(\bar{x}_i)$.

Assumption 4 (See [32]). Assume that there exist the constants $K_{c_i}^0$, $i = 1, \ldots, n$, and $K_{c_i}^j$, $j = 1, \ldots, n$, such that $k_{c_i} \leqslant K_{c_i}^0$ and $k_{c_i}^{(j)} \leqslant K_{c_i}^j$, $\forall t \geqslant 0$.

Lemma 1 ([46, 47]). The Lyapunov function candidate $V(t)$ is bounded given that the initial condition $V(0)$ is bounded, $V(t) \geqslant 0$ is continuous and $\dot{V}(t) \leqslant -\lambda V(t) + \mu$, where $\lambda > 0$ and $\mu > 0$.

Lemma 2 (See [29]). If the condition $|x_i| < k_{c_i}, i = 1, \ldots, n, \forall t \geqslant 0$, V_i^* are satisfied, then the following inequality holds:

$$V_i^* \leqslant \frac{e_i^2 k_{c_i}^2}{k_{c_i}^2 - x_i^2}.$$

3 The adaptive control and stability analysis

3.1 Adaptive tracking control

Considering the intelligent modeling scheme and the backstepping method, this subsection gives the detailed design of the adaptive laws and control law with full state constraints using iBLFs.

Step 1. The tracking error is defined as $e_1 = x_1 - y_d$. Then, its derivative is

$$\dot{e}_1 = f_{1,k}\left(x_1\right) + g_1\left(x_1\right)x_2 - \dot{y}_d. \tag{2}$$

In order to ensure the states satisfy the constraints, the following integral-type Lyapunov function candidate is selected:

$$V_1^* = \int_0^{e_1} \frac{\xi k_{c_1}^2}{k_{c_1}^2 - \left(\xi + y_d\right)^2}\mathrm{d}\xi, \tag{3}$$

where $\left|y_d\left(t\right)\right| \leqslant Y_0 < k_{c_1}$. It can be seen that V_1^* is positive definite, continuously differentiable, and it also satisfies the decrescent condition in the interval $\left|x_1\right| < k_{c_1}$. Then, the following inequality holds:

$$\frac{1}{2}e_1^2 \leqslant V_1^* \leqslant e_1^2 \int_0^1 \frac{\omega k_{c_1}^2}{k_{c_1}^2 - \left(\omega e_1 + \mathrm{sgn}\left(e_1\right)Y_0\left(t\right)\right)^2}\mathrm{d}\omega, \tag{4}$$

where the substitution $\xi = \omega e_1$ will be utilized in the following recursive formulas.

Remark 2. As shown in (3) and (4), BLF, an integral type, is used to settle the state constraints. Compared with the logarithmic and tangential BLFs, the integral BLFs can eliminate the conservative restriction of replacing the state constraint with the error constraint, i.e., the direct constraint on the state variable can be realized.

The time derivative is written as

$$\dot{V}_1^* = \frac{k_{c_1}^2 e_1}{k_{c_1}^2 - x_1^2}\dot{e}_1 + \frac{\partial V_1^*}{\partial y_d}\dot{y}_d, \tag{5}$$

where

$$\frac{\partial V_1^*}{\partial y_d} = e_1\left(\frac{k_{c_1}^2}{k_{c_1}^2 - x_1^2} - \Phi_1\left(e_1, y_d\right)\right), \tag{6}$$

with

$$\begin{aligned}\Phi_1\left(e_1, y_d\right) &= \int_0^1 \frac{k_{c_1}^2}{k_{c_1}^2 - \left(\omega e_1 + y_d\right)^2}\mathrm{d}\omega,\\ &= \frac{k_{c_1}}{e_1}\left(\tanh^{-1}\left(\frac{e_1 + y_d}{k_{c_1}}\right) - \tanh^{-1}\left(\frac{y_d}{k_{c_1}}\right)\right)\\ &= \frac{k_{c_1}}{2e_1}\ln\frac{\left(k_{c_1} + x_1\right)\left(k_{c_1} - y_d\right)}{\left(k_{c_1} - x_1\right)\left(k_{c_1} + y_d\right)}.\end{aligned} \tag{7}$$

Remark 3. The iBLF belongs to variable limit integral (VLI). In other words, the integrand in the VLI is related with a binary function. In the process of solving the integral issue, the designers exploit the variable substitution method to convert it into a simple function. For the sake of avoiding the binary function, its partial derivatives are introduced into the monadic calculus. Thus, the above derivative formula of VLI is realized using the knowledge of variable calculus.

Remark 4. According to Assumption 1, considering the function $\Phi_1\left(e_1, y_d\right)$, we know that it satisfies the condition of L'Hospital's rule. Thus, we obtain

$$\lim_{e_1 \to 0}\Phi_1\left(e_1, y_d\right) = \frac{k_{c_1}^2}{k_{c_1}^2 - {y_d}^2}, \tag{8}$$

where $\Phi_1\left(e_1, y_d\right)$ is well defined in a neighborhood of $e_1 = 0$ [29].

Let $e_2 = x_2 - \alpha_1$. Hence, the derivative of V_1^* is rewritten as

$$\dot{V}_1^* = \frac{e_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2}\left[f_{1,k}\left(x_1\right) + g_1\left(x_1\right)e_2 + g_{1,k}\left(x_1\right)\alpha_1\right] - e_1\Phi_1\left(e_1, y_d\right)\dot{y}_d. \tag{9}$$

Define

$$H_{1,k}\left(X_1\right)=f_{1,k}\left(x_1\right)-\frac{k_{c_1}^2-x_1^2}{k_{c_1}^2}\Phi_1\left(e_1,y_d\right)\dot{y}_d, \tag{10}$$

where $X_1=\left[x_1,y_d,\dot{y}_d\right]^{\mathrm{T}}$. Because the unknown function $H_{1,k}\left(X_1\right)$ is unobtainable, the FLS is used. $H_{1,k}\left(X_1\right)$ is approximated as

$$H_{1,k}\left(X_1\right)=\theta_{1,k}^{\mathrm{T}}\varphi_1\left(X_1\right)+\varepsilon_{1,k}\left(X_1\right),\quad \left|\varepsilon_{1,k}\left(X_1\right)\right|\leqslant\bar{\varepsilon}_1. \tag{11}$$

Further, the derivative of (9) is described as

$$\begin{aligned}\dot{V}_1^*=&\frac{e_1k_{c_1}^2}{k_{c_1}^2-x_1^2}\left(\theta_{1,k}^{\mathrm{T}}\varphi_1\left(X_1\right)+\varepsilon_{1,k}\left(X_1\right)\right)\\&+\frac{e_1k_{c_1}^2}{k_{c_1}^2-x_1^2}\left(g_1\left(x_1\right)e_2+g_1\left(x_1\right)\alpha_1\right).\end{aligned} \tag{12}$$

The following Lyapunov function candidate is designed:

$$V_1=V_1^*+\frac{1}{2\gamma_1}\tilde{\Theta}_1^2, \tag{13}$$

where $\tilde{\Theta}_1=\Theta_1-\hat{\Theta}_1$ represents the estimate error, in which $\hat{\Theta}_1$ stands for the estimation of Θ_1. γ_1 is a positive constant. Then, it results in

$$\begin{aligned}\dot{V}_1=&\frac{e_1k_{c_1}^2}{k_{c_1}^2-x_1^2}\left(\theta_{1,k}^{\mathrm{T}}\varphi_1\left(X_1\right)+\varepsilon_{1,k}\left(X_1\right)\right)\\&+\frac{e_1k_{c_1}^2}{k_{c_1}^2-x_1^2}\left(g_1\left(x_1\right)e_2+g_1\left(x_1\right)\alpha_1\right)-\frac{1}{\gamma_1}\tilde{\Theta}_1\dot{\hat{\Theta}}_1.\end{aligned} \tag{14}$$

Based on Young's inequality, we obtain

$$\begin{aligned}\frac{e_1k_{c_1}^2}{k_{c_1}^2-x_1^2}\theta_{1,k}^{\mathrm{T}}\varphi_1\left(X_1\right)\leqslant&\frac{\|\theta_{1,k}\|^2\varphi_1^{\mathrm{T}}\left(X_1\right)\varphi_1\left(X_1\right)}{2\mu_{1,k}^2}\left(\frac{e_1k_{c_1}^2}{k_{c_1}^2-x_1^2}\right)^2+\frac{\mu_{1,k}^2}{2}\\\leqslant&\frac{\Theta_1\varphi_1^{\mathrm{T}}\left(X_1\right)\varphi_1\left(X_1\right)}{2\mu_{1,k}^2}\left(\frac{e_1k_{c_1}^2}{k_{c_1}^2-x_1^2}\right)^2+\frac{\mu_{1,k}^2}{2},\end{aligned} \tag{15}$$

and

$$\begin{aligned}\frac{e_1k_{c_1}^2}{k_{c_1}^2-x_1^2}\varepsilon_{1,k}\left(X_1\right)\leqslant&\frac{1}{2}\left(\frac{e_1k_{c_1}^2}{k_{c_1}^2-x_1^2}\right)^2+\frac{1}{2}\varepsilon_{1,k}^2\left(X_1\right)\\\leqslant&\frac{1}{2}\left(\frac{e_1k_{c_1}^2}{k_{c_1}^2-x_1^2}\right)^2+\frac{1}{2}\bar{\varepsilon}_1^2,\end{aligned} \tag{16}$$

where $\Theta_1=\max\|\theta_{1,k}\|^2$, $k\in\Gamma$. $\mu_{1,k}>0$ is a constant.

Remark 5. In order to construct a common coordinate transform, we define $\Theta_i=\max\{\|\theta_{i,k}\|^2,k\in\Gamma\}$ (see [48, 49]). This paper adopts arbitrary switching rule which should depend on the common Lyapunov function. In the recursive derivation, it is essential to build a common coordinate transform for constructing a common Lyapunov function. Because switching systems contain multiple subsystems, they are different from each other, and then they will affect the common coordinate transforms. As a result, the construction of common Lyapunov function are hard to achieve. Therefore, the definition of $\Theta_i=\max\{\|\theta_{i,k}\|^2,k\in\Gamma\}$ is extremely important, which is useful when the common Lyapunov function is established.

We design the virtual controller α_1 as

$$\alpha_1 = \frac{1}{\underline{g}_1}\left[-K_1 e_1 - \frac{\hat{\Theta}_1 \varphi_1^{\mathrm{T}}(X_1)\varphi_1(X_1)}{2\mu_{1,\min}^2}\frac{e_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2} - \frac{1}{2}\frac{e_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2}\right], \tag{17}$$

where $K_1 > 0$ is designed by the users. $\mu_{1,\min} = \min\{\mu_{1,k}, k \in \Gamma\}$. It is essential to notice that α_1 is a function of x_1, y_d, $\dot{y}_d$ and $\hat{\Theta}_1$.

The update law is established as

$$\dot{\hat{\Theta}}_1 = \frac{\gamma_1 \varphi_1^{\mathrm{T}}(X_1)\varphi_1(X_1)}{2\mu_{1,\min}^2}\left(\frac{e_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2}\right)^2 - \beta_1 \hat{\Theta}_1. \tag{18}$$

Because $\tilde{\Theta}_1 \hat{\Theta}_1 \leqslant -\frac{1}{2}\tilde{\Theta}_1^2 + \frac{1}{2}\Theta_1^2$, the first-order derivative of V_1 is

$$\begin{aligned}\dot{V}_1 \leqslant & -\frac{K_1 e_1^2 k_{c_1}^2}{k_{c_1}^2 - x_1^2} - \frac{\beta_1}{2\gamma_1}\tilde{\Theta}_1^2 + g_1(x_1)\, e_2 \frac{e_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2} \\ & + \frac{\beta_1}{2\gamma_1}\Theta_1^2 + \frac{1}{2}\mu_{1,\max}^2 + \frac{1}{2}\bar{\varepsilon}_1^2,\end{aligned} \tag{19}$$

with $\mu_{1,\max} = \max\{\mu_{1,k}, k \in \Gamma\}$.

Step i ($2 \leqslant i \leqslant n-1$). Define the error $z_i = x_i - \alpha_{i-1}$, and its time derivative is given as

$$\dot{z}_i = f_{i,k}(\bar{x}_i) + g_i(\bar{x}_i)\, x_{i+1} - \dot{\alpha}_{i-1}, \tag{20}$$

where α_{i-1} is the functions of $\bar{x}_{i-1}, y_d, \ldots, y_d^{(i-1)}, \hat{\Theta}_1, \ldots, \hat{\Theta}_{i-1}$. Then, the time derivative of α_{i-1} is

$$\dot{\alpha}_{i-1} = \sum_{m=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_m}\dot{x}_m + \sum_{m=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial \hat{\Theta}_m}\dot{\hat{\Theta}}_m + \sum_{m=0}^{i-1}\frac{\partial \alpha_{i-1}}{\partial y_d^{(m)}}y_d^{(m+1)}. \tag{21}$$

The integral-type Lyapunov function candidate is considered:

$$V_i^* = \int_0^{e_i}\frac{\xi k_{c_i}^2}{k_{c_i}^2 - (\xi + \alpha_{i-1})^2}\mathrm{d}\xi, \tag{22}$$

with $\xi = \omega e_i$ and the following inequality holds:

$$\frac{1}{2}e_i^2 \leqslant V_i^* \leqslant e_i^2 \int_0^1 \frac{\omega k_{c_i}^2}{k_{c_i}^2 - (\omega e_i + \operatorname{sgn}(e_i)\, A_{i-1})^2}\mathrm{d}\omega. \tag{23}$$

The virtual controllers $\alpha_i, \ldots, \alpha_{n-1}$ are continuously differentiable functions and satisfy $|\alpha_{i-1}| \leqslant A_{i-1} < k_{c_i}$, where $A_{i-1}, i = 2, \ldots, n$ are constants decided by designers. Similar to Step 1, the first order derivative of V_i^* is

$$\dot{V}_i^* = \frac{e_i k_{c_i}^2}{k_{c_i}^2 - x_i^2}\dot{e}_i + \frac{\partial V_i^*}{\partial \alpha_{i-1}}\dot{\alpha}_{i-1}, \tag{24}$$

where

$$\frac{\partial V_i^*}{\partial \alpha_{i-1}} = e_i\left(\frac{k_{c_i}^2}{k_{c_i}^2 - x_i^2} - \Phi_i(e_i, \alpha_{i-1})\right), \tag{25}$$

$$\begin{aligned}\Phi_i(e_i, \alpha_{i-1}) &= \int_0^1 \frac{k_{c_i}^2}{k_{c_i}^2 - (\omega e_i + \alpha_{i-1})^2}\mathrm{d}\omega \\ &= \frac{k_{c_i}}{e_i}\left(\tanh^{-1}\left(\frac{e_i + \alpha_{i-1}}{k_{c_i}}\right) - \tanh^{-1}\left(\frac{\alpha_{i-1}}{k_{c_i}}\right)\right)\end{aligned}$$

$$= \frac{k_{c_i}}{2e_i} \ln \frac{(k_{c_i}+x_i)(k_{c_i}-\alpha_{i-1})}{(k_{c_i}-x_i)(k_{c_i}+\alpha_{i-1})}, \tag{26}$$

with $\Phi_i(e_i,\alpha_{i-1})$ being well defined in a neighborhood of $e_i=0$ [29].

According to the above analysis, the following equation is obtained:

$$\dot{V}_i^* = \frac{e_i k_{c_i}^2}{k_{c_i}^2 - x_i^2}\left[f_{i,k}(\bar{x}_i) + g_i(\bar{x}_i)e_i + g_i(\bar{x}_i)\alpha_{i-1}\right] - e_i\Phi_i(e_i,\alpha_{i-1})\dot{\alpha}_{i-1}. \tag{27}$$

We define unknown nonlinear functions $H_{i,k}(X_i)$ as

$$\begin{aligned} H_{i,k}(X_i) &= f_{i,k}(\bar{x}_i) - \frac{k_{c_i}^2 - x_i^2}{k_{c_i}^2}\Phi_i(e_i,\alpha_{i-1})\dot{\alpha}_{i-1} \\ &\quad + g_{i-1}(t)e_{i-1}e_i\frac{k_{c_{i-1}}^2}{k_{c_{i-1}}^2 - x_{i-1}^2} \\ &= \theta_{i,k}^{\mathrm{T}}\varphi_i(X_i) + \varepsilon_{i,k}(X_i), \quad |\varepsilon_{i,k}(X_i)| \leqslant \bar{\varepsilon}_i, \end{aligned} \tag{28}$$

where $X_i = [\bar{x}_i, y_d, \ldots, y_d^{(i)}, \hat{\Theta}_1, \ldots, \hat{\Theta}_{i-1}]^{\mathrm{T}}$.

We choose the following Lyapunov function:

$$V_i = V_{i-1} + V_i^* + \frac{1}{2\gamma_i}\tilde{\Theta}_i^2, \tag{29}$$

with $\gamma_i > 0$ being a parameter.

The first order derivative of (29) is deduced as

$$\begin{aligned} \dot{V}_i &= \dot{V}_{i-1} + \frac{e_i k_{c_i}^2}{k_{c_i}^2 - x_i^2}\left(\theta_{i,k}^{\mathrm{T}}\varphi_i(X_i) + \varepsilon_{i,k}(X_i)\right) \\ &\quad + \frac{e_i k_{c_i}^2}{k_{c_i}^2 - x_i^2}\left(g_i(\bar{x}_i)e_{i+1} + g_i(\bar{x}_i)\alpha_i\right) - \frac{1}{\gamma_i}\tilde{\Theta}_i\dot{\hat{\Theta}}_i. \end{aligned} \tag{30}$$

Based on Young's inequality, we get that

$$\begin{aligned} \frac{e_i k_{ci}^2}{k_{c_i}^2 - x_i^2}\theta_{i,k}^{\mathrm{T}}\varphi_i(X_i) &\leqslant \frac{\|\theta_{i,k}\|^2\varphi_i^{\mathrm{T}}(X_i)\varphi_i(X_i)}{2\mu_{i,k}^2}\left(\frac{e_i k_{c_i}^2}{k_{c_i}^2 - x_i^2}\right)^2 + \frac{\mu_{i,k}^2}{2} \\ &\leqslant \frac{\Theta_i\varphi_i^{\mathrm{T}}(X_i)\varphi_i(X_i)}{2\mu_{i,k}^2}\left(\frac{e_i k_{c_i}^2}{k_{c_i}^2 - x_i^2}\right)^2 + \frac{\mu_{i,k}^2}{2}, \end{aligned} \tag{31}$$

and

$$\begin{aligned} \frac{e_i k_{c_i}^2}{k_{c_i}^2 - x_i^2}\varepsilon_{i,k}(X_i) &\leqslant \frac{1}{2}\left(\frac{e_i k_{c_i}^2}{k_{c_i}^2 - x_i^2}\right)^2 + \frac{1}{2}\varepsilon_{i,k}^2(X_i) \\ &\leqslant \frac{1}{2}\left(\frac{e_i k_{c_i}^2}{k_{c_i}^2 - x_i^2}\right)^2 + \frac{1}{2}\bar{\varepsilon}_i^2, \end{aligned} \tag{32}$$

where $\Theta_i = \max\|\theta_{i,k}\|^2, k \in \Gamma$. $\mu_{i,k}$ is a positive parameter.

Here, the virtual controllers α_i are constructed in the following form:

$$\alpha_i = \frac{1}{\underline{g}_i}\left[-K_i e_i - \frac{\hat{\Theta}_i\varphi_i^{\mathrm{T}}(X_i)\varphi_i(X_i)}{2\mu_{i,\min}^2}\frac{e_i k_{c_i}^2}{k_{c_i}^2 - x_i^2} - \frac{1}{2}\frac{e_i k_{c_i}^2}{k_{c_i}^2 - x_i^2}\right], \tag{33}$$

where $K_i > 0$ is designed by the users, and $\mu_{i,\min} = \min\{\mu_{i,k}, k \in \Gamma\}$.

We develop the update laws as

$$\dot{\hat{\Theta}}_i = \frac{\gamma_i\varphi_i^{\mathrm{T}}(X_i)\varphi_i(X_i)}{2\mu_{i,\min}^2}\left(\frac{e_i k_{c_i}^2}{k_{c_i}^2 - x_i^2}\right)^2 - \beta_i\hat{\Theta}_i. \tag{34}$$

Because $\tilde{\Theta}_i\hat{\Theta}_i \leqslant -\frac{1}{2}\tilde{\Theta}_i^2 + \frac{1}{2}\Theta_i^2$, the first order derivative of (30) is deduced as

$$\begin{aligned}\dot{V}_i \leqslant &- \sum_{m=1}^{i} \frac{K_m e_m^2 k_{c_m}^2}{k_{c_m}^2 - x_m^2} - \sum_{m=1}^{i} \frac{\beta_m}{2\gamma_m}\tilde{\Theta}_m^2 + g_m(\bar{x}_m) e_m e_{m+1} \frac{k_{c_m}^2}{k_{c_m}^2 - x_m^2} \\ &+ \sum_{m=1}^{i} \left(\frac{\beta_m}{2\gamma_m}\Theta_m^2 + \frac{1}{2}\bar{\varepsilon}_m^2 + \frac{1}{2}\mu_{m,\max}^2 \right). \end{aligned} \tag{35}$$

Step n. Let $z_n = x_n - \alpha_{n-1}$. Thus, we get

$$\dot{z}_n = f_{n.k}(\bar{x}_n) + g_n(\bar{x}_n) u - \dot{\alpha}_{n-1}. \tag{36}$$

Then, $\dot{\alpha}_{n-1}$ is given by

$$\dot{\alpha}_{n-1} = \sum_{m=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_m}\dot{x}_m + \sum_{m=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\xi}_m}\dot{\hat{\Theta}}_m + \sum_{m=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(m)}} y_d^{(m+1)}. \tag{37}$$

Consider the integral-type Lyapunov function candidate:

$$V_n^* = \int_0^{e_n} \frac{\xi k_{c_n}^2}{k_{c_n}^2 - (\xi + \alpha_{n-1})^2} \mathrm{d}\xi, \tag{38}$$

where $\xi = \omega e_n$. Hence, the following inequality holds:

$$\frac{1}{2}e_n^2 \leqslant V_n^* \leqslant e_n^2 \int_0^1 \frac{\omega k_{c_n}^2}{k_{c_n}^2 - (\omega e_n + \mathrm{sgn}(e_n) A_{n-1})^2} \mathrm{d}\omega. \tag{39}$$

Similar to Step i, it holds that

$$\dot{V}_n^* = \frac{e_n k_{c_n}^2}{k_{c_n}^2 - x_n^2}\dot{e}_n + \frac{\partial V_n^*}{\partial \alpha_{n-1}}\dot{\alpha}_{n-1}, \tag{40}$$

where

$$\frac{\partial V_n^*}{\partial \alpha_{n-1}} = e_n \left(\frac{k_{c_n}^2}{k_{c_n}^2 - x_n^2} - \Phi_n(e_n, \alpha_{n-1}) \right), \tag{41}$$

with

$$\begin{aligned}\Phi_n(e_n, \alpha_{n-1}) &= \int_0^1 \frac{k_{c_n}^2}{k_{c_n}^2 - (\omega z_n + \alpha_{n-1})^2} \mathrm{d}\omega \\ &= \frac{k_{c_n}}{e_n} \left(\tanh^{-1}\left(\frac{e_n + \alpha_{n-1}}{k_{c_n}} \right) - \tanh^{-1}\left(\frac{\alpha_{n-1}}{k_{c_n}} \right) \right) \\ &= \frac{k_{c_n}}{2e_n} \ln \frac{(k_{c_n} + x_n)(k_{c_n} - \alpha_{n-1})}{(k_{c_n} - x_n)(k_{c_n} + \alpha_{n-1})}. \end{aligned} \tag{42}$$

Then, referring to Remark 4, $\Phi_n(e_n, \alpha_{n-1})$ is well defined to be bounded in a neighborhood of $e_n = 0$. Moreover, it leads to

$$\dot{V}_n^* = \frac{e_n k_{c_n}^2}{k_{c_n}^2 - x_n^2}(f_{n,k}(\bar{x}_n) + g_n(\bar{x}_n) u) - e_n \Phi_n(e_n, \alpha_{n-1}) \dot{\alpha}_{n-1}. \tag{43}$$

We define the unknown nonlinear function as

$$\begin{aligned}H_{n,k}(X_n) = f_{n,k}(\bar{x}_n) - \frac{k_{c_n}^2 - x_n^2}{k_{c_n}^2}\Phi_n(e_n, \alpha_{n-1}) \dot{\alpha}_{i-1} \\ + g_{n-1}(t) e_{n-1} e_n \frac{k_{c_{n-1}}^2}{k_{c_{n-1}}^2 - x_{n-1}^2}\end{aligned}$$

$$= \theta_{n,k}^{\mathrm{T}} \varphi_n (X_n) + \varepsilon_{n,k} (X_n), \quad |\varepsilon_{n,k} (X_i)| \leqslant \bar{\varepsilon}_n, \tag{44}$$

where $X_i = [\bar{x}_n, y_d, \ldots, y_d^{(n)}, \hat{\Theta}_1, \ldots, \hat{\Theta}_n]^{\mathrm{T}}$.

Design the following Lyapunov function:

$$V_n = V_{n-1} + V_n^* + \frac{1}{2\gamma_n} \tilde{\Theta}_n^2, \tag{45}$$

where γ_n is a positive parameter.

Next, it results in

$$\begin{aligned} \dot{V}_n = \dot{V}_{n-1} &+ \frac{e_n k_{c_n}^2}{k_{c_n}^2 - x_n^2} \left(\theta_{n,k}^{\mathrm{T}} \varphi_n (X_n) + \varepsilon_{n,k} (X_n) \right) \\ &+ \frac{e_n k_{c_n}^2}{k_{c_n}^2 - x_n^2} g_n (\bar{x}_n) u - \frac{1}{\gamma_n} \tilde{\Theta}_n \dot{\hat{\Theta}}_n. \end{aligned} \tag{46}$$

Based on Young's inequality, we obtain

$$\begin{aligned} \frac{e_n k_{c_n}^2}{k_{c_n}^2 - x_n^2} \theta_{n,k}^{\mathrm{T}} \varphi_n (X_n) &\leqslant \frac{\|\theta_{n,k}\|^2 \varphi_n^{\mathrm{T}} (X_n) \varphi_n (X_n)}{2\mu_{n,k}^2} \left(\frac{e_n k_{c_n}^2}{k_{c_n}^2 - x_n^2} \right)^2 + \frac{\mu_{n,k}^2}{2} \\ &\leqslant \frac{\Theta_n \varphi_n^{\mathrm{T}} (X_n) \varphi_n (X_n)}{2\mu_{n,k}^2} \left(\frac{e_n k_{c_n}^2}{k_{c_n}^2 - x_n^2} \right)^2 + \frac{\mu_{n,k}^2}{2}, \end{aligned} \tag{47}$$

and

$$\begin{aligned} \frac{e_n k_{c_n}^2}{k_{c_n}^2 - x_n^2} \varepsilon_{n,k} (X_n) &\leqslant \frac{1}{2} \left(\frac{e_n k_{c_n}^2}{k_{c_n}^2 - x_n^2} \right)^2 + \frac{1}{2} \varepsilon_{n,k}^2 (X_n) \\ &\leqslant \frac{1}{2} \left(\frac{e_n k_{c_n}^2}{k_{c_n}^2 - x_n^2} \right)^2 + \frac{1}{2} \bar{\varepsilon}_n^2, \end{aligned} \tag{48}$$

where $\Theta_n = \max \|\theta_{n,k}\|^2, k \in \Gamma$. $\mu_{n,k}$ is a positive constant.

The actual controller in this paper is set up as

$$u = \frac{1}{\underline{g}_n} \left[-K_n e_n - \frac{\hat{\Theta}_n \varphi_n^{\mathrm{T}} (X_n) \varphi_n (X_n)}{2\mu_{n,\min}^2} \frac{e_n k_{c_n}^2}{k_{c_n}^2 - x_n^2} - \frac{1}{2} \frac{e_n k_{c_n}^2}{k_{c_n}^2 - x_n^2} \right], \tag{49}$$

where $K_n > 0$ is decided by designers, $\mu_{n,\min} = \min\{\mu_{n,k}, k \in \Gamma\}$.

The tuning law is proposed as

$$\dot{\hat{\Theta}}_n = \frac{\gamma_n \varphi_n^{\mathrm{T}} (X_n) \varphi_n (X_n)}{2\mu_{n,\min}^2} \left(\frac{e_n k_{c_n}^2}{k_{c_n}^2 - x_n^2} \right)^2 - \beta_n \hat{\Theta}_n. \tag{50}$$

Based on the fact that $\tilde{\Theta}_n \hat{\Theta}_n \leqslant -\frac{1}{2} \tilde{\Theta}_n^2 + \frac{1}{2} \Theta_n^2$, we get the first order derivative of (45) as

$$\dot{V}_n \leqslant - \sum_{m=1}^{n} \frac{K_m e_m^2 k_{c_m}^2}{k_{c_m}^2 - x_m^2} - \sum_{m=1}^{n} \frac{\beta_m}{2\gamma_m} \tilde{\Theta}_m^2 + \sum_{m=1}^{n} \left(\frac{\beta_m}{2\gamma_m} \Theta_m^2 + \frac{1}{2} \bar{\varepsilon}_m^2 + \frac{1}{2} \mu_{m,\max}^2 \right), \tag{51}$$

with $\mu_{n,\max} = \max\{\mu_{n,k}, k \in \Gamma\}$.

The main results for our proposed method are concluded in Subsection 3.2.

3.2 Theorem and proof

Theorem 1. Considering the uncertain switched nonlinear system (1), under Assumptions 1–4, construct the virtual controllers (17) and (33), the adaptive controller (49), the adaptive laws (18), (34) and (50). If the initial states satisfy $|x_i(0)| < |k_{c_i}|$, then the whole adaptive control scheme ensures that all

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the signals in the resulted system are SGUUB and all the state constraints are guaranteed under arbitrary switchings. Moreover, the errors remain in the corresponding compact sets.

Proof. (1) All the signals appearing in the design are SGUUB. Then, select the common Lyapunov function as

$$V\left(t\right)=V_{n}=\sum_{m=1}^{n}\int_{0}^{e_{m}}\frac{\xi k_{c_{m}}^{2}}{k_{c_{m}}^{2}-\left(\xi+\alpha_{m-1}\right)^{2}}\mathrm{d}\xi+\frac{1}{2}\sum_{m=1}^{n}\frac{1}{\gamma_{m}}\tilde{\Theta}_{m}^{2}, \tag{52}$$

where $\alpha_0 = y_d$. According to Lemma 2, we get

$$V\left(t\right)\leqslant\sum_{m=1}^{n}\frac{k_{c_{m}}^{2}e_{m}^{2}}{k_{c_{m}}^{2}-x_{m}^{2}}+\frac{1}{2}\sum_{m=1}^{n}\frac{1}{\gamma_{m}}\tilde{\Theta}_{m}^{2}. \tag{53}$$

Based on the above mentioned analysis, it holds that

$$\dot{V}\left(t\right)\leqslant-\eta V\left(t\right)+C, \tag{54}$$

where $\eta=\min\left\{K_{m},\beta_{m},m=1,2,\ldots,n\right\}$ and $C=\frac{1}{2}\sum_{m=1}^{n}(\frac{\beta_{m}}{\gamma_{m}}\Theta_{m}^{2}+\bar{\varepsilon}_{m}^{2}+\mu_{m,\max}^{2})$. Multiplying both sides of (54) by $\mathrm{e}^{\lambda t}$, and integrating it over $[0,t]$, we have

$$V_{n}\left(t\right)\leqslant\mathrm{e}^{-\lambda t}\left(V_{n}\left(0\right)-C/\lambda\right)+C/\lambda. \tag{55}$$

On the strength of Lemma 1, inequality (54), definition of $V(t)$, and Lyapunov stability theorem [21], all the signals in the resulted system are SGUUB.

(2) Full state constraints are guaranteed. To make sure that states cannot violate constraints, we assume that there exist some $t = T$ and $m \in \{1,\ldots,n\}$ such that $|x_m(T)| = k_{c_m}$. Then, the following inequality is obtained:

$$\begin{aligned}V_{n}|_{t=T}&=\sum_{m=1}^{n}V_{m}^{*}|_{t=T}+\sum_{m=1}^{n}V_{m}^{\Theta}\big|_{t=T}\\&=\sum_{m=1}^{n}\int_{0}^{e_{m}(T)}\frac{\xi k_{c_{m}}^{2}}{k_{c_{m}}^{2}-\left(\xi+\alpha_{m-1}\right)^{2}}\mathrm{d}\xi+\frac{1}{2}\sum_{m=1}^{n}\tilde{\Theta}_{m}^{2}\Big|_{t=T}\\&\leqslant\sum_{m=1}^{n}V_{m}^{*}|_{t=0}+\frac{1}{2}\sum_{m=1}^{n}\tilde{\Theta}_{m}^{2}\Big|_{t=T}.\end{aligned} \tag{56}$$

Integrating $V_m^*|_{t=T}, m=1,\ldots,n$, we obtain

$$\begin{aligned}V_{m}^{*}|_{t=T}&=k_{c_{m}}\left(\xi\tanh^{-1}\frac{\xi+\alpha_{m-1}}{k_{c_{m}}}\right)\Big|_{0}^{e_{m}}-k_{c_{m}}\int_{0}^{e_{m}}\tanh^{-1}\frac{\xi+\alpha_{m-1}}{k_{c_{m}}}\mathrm{d}\xi\\&=k_{c_{m}}\alpha_{m-1}\left(T\right)\ln\frac{\left(1+\alpha_{m-1}\left(T\right)\right)\left(1-x_{m}\left(T\right)\right)}{\left(1-\alpha_{m-1}\left(T\right)\right)\left(1+x_{m}\left(T\right)\right)}\\&\quad+\frac{k_{c_{m}}^{2}}{2}\ln\frac{\left(k_{c_{m}}^{2}-\alpha_{m-1}^{2}\left(T\right)\right)}{\left(k_{c_{m}}^{2}-x_{m}^{2}\left(T\right)\right)}\leqslant V_{m}^{*}|_{t=0},\end{aligned} \tag{57}$$

while $|x_m(T)| = k_{c_m}$. $V_m^*|_{t=T}$ becomes unbounded and contradict with $V_m^*|_{t=T} \leqslant V_m^*|_{t=0}, \forall t \geqslant 0$. Therefore, $|x_m(T)| \neq k_{c_m}$ and for the giving initial state $x_m(0) \in \{x_m \in \mathbb{R} | \, |x_m| < k_{c_m}\}$ it is clear that $|x_m(T)| < k_{c_m}$.

(3) The errors e_m remain in a compact set, which is defined by

$$\Omega_{z}=\left\{e_{m}\leqslant\Gamma,\Gamma=\sqrt{2V\left(0\right)\mathrm{e}^{-\lambda t}+2C/\lambda}\right\}.$$

Because $e_m^2 \leqslant 2V(t) \leqslant 2\mathrm{e}^{-\lambda t}(V(0)-C/\lambda)+2C/\lambda \leqslant 2C/\lambda + 2V(0)\mathrm{e}^{-\lambda t}$, we have

$$|e_{m}|\leqslant\sqrt{2V\left(0\right)\mathrm{e}^{-\lambda t}+2C/\lambda}. \tag{58}$$

Especially, we get $|e_1| \leqslant \sqrt{2\mathrm{e}^{-\lambda t}\left(V\left(0\right)-C/\lambda\right)+2C/\lambda}$. If the initial condition satisfies $V\left(0\right)=C/\lambda$, it is clear that $|z_1| \leqslant \sqrt{2C/\lambda}$. If the condition could be hardly achieved, there exists T such that for any $t > T$, the result is still satisfied. Thus, e_1 can be arbitrarily small by choosing the appropriate design parameters.

Consequently, we have finished the proof of Theorem 1.

Remark 6. In contrast to the existing control methods for the switched systems, there are two aspects to be noted. (1) The constraint control problem for strict feedback switched nonlinear systems is solved in this paper, but it is ignored in [10, 35, 36, 50]. (2) Although the output/state constraint issues are handled in [39, 41, 43] for various switched nonlinear systems, they are all obtained though log-BLFs or tan-BLFs, which are indirect schemes. The developed iBLF-based control strategy is a direct constraint scheme.

4 Simulation example

In order to illustrate the effectiveness of the developed approach, the switched nonlinear system is studied, and it contains two subsystems as

$$\begin{cases} \dot{x}_1 = 0.5\sin(x_1^2) + (0.2 + \exp^{0.1x_1})x_2, \\ \dot{x}_2 = 0.1(x_1+x_2)^2 + (0.3\cos(0.4x_1x_2))u, \\ y = x_1, \end{cases} \tag{59}$$

$$\begin{cases} \dot{x}_1 = x_1x_2 + x_1 + (0.2 + \exp^{0.1x_1})x_2, \\ \dot{x}_2 = \cos(x_1) + (0.3\cos(0.4x_1x_2))u, \\ y = x_1, \end{cases} \tag{60}$$

where x_1 and x_2 are system states, which are constrained by $|x_1| < k_{c_1} = 0.41$ and $|x_2| < k_{c_2} = 0.7$, respectively.

u is the system input designed as follows:

$$u = \frac{1}{\underline{g}_2}\left[-\left(K_2+\bar{K}_2\left(t\right)\right)e_2 - \frac{\hat{\Theta}_2\varphi_2^{\mathrm{T}}\left(X_2\right)\varphi_2\left(X_2\right)}{2\mu_{2,\min}^2}\frac{e_2k_{c_2}^2\left(t\right)}{k_{c_2}^2\left(t\right)-x_2^2} - \frac{1}{2}\frac{e_2k_{c_2}^2\left(t\right)}{k_{c_2}^2\left(t\right)-x_2^2}\right],$$

with $K_2 = 20$, $\mu_{2,\min} = 1.2$. y denotes the system output, which is required to track the reference signal $y_d = 0.25\sin(t)$.

In this simulation, the initial values of system states are selected as $x_1(0) = x_2(0) = 0$, and the initial values of adaptive laws are chosen as $\hat{\Theta}_1(0) = 0.5$, $\hat{\Theta}_2(0) = 0.1$. In addition, the related parameters are $K_1 = 25$, $\gamma_1 = 0.4$, $\gamma_2 = 0.5$, $\beta_1 = 0.8$, $\beta_2 = 0.3$, $\mu_{1,\min} = 1.5$. The switching time is designed as follows:

$$\sigma = \begin{cases} 1, & t \in [0,4) \cup [15,36) \cup [68,82), \\ 2, & t \in [4,15) \cup [36,68) \cup [82,100). \end{cases} \tag{61}$$

Figures 1–6 express the simulation results. Figure 1 describes the switching signal. Figure 2 gives the trajectories of system state x_1, reference signal y_d, and the constraint bounds of x_1. From Figure 2, we conclude that the system state x_1 can well track the reference signal y_d. At the same time, the constraint condition of x_1 is also satisfied. Figure 3 describes the curves of x_2 and its constraint bounds, which illustrates that the constraint condition is not violated. Figure 4 is the phase diagram of e_1 and e_2. Figure 5 gives the trajectories of adaptive laws. And the curves of system input u are shown in Figure 6. It can be seen from these figures that all the signals in the closed-loop systems are all bounded.

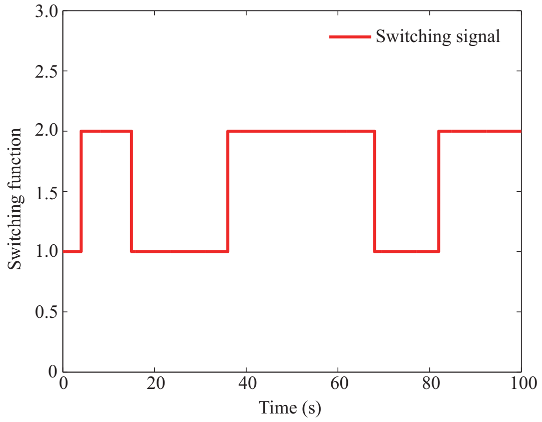

Figure 1 (Color online) The switching signal.

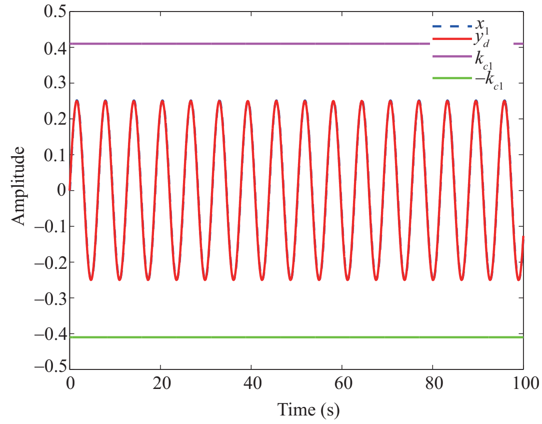

Figure 2 (Color online) The trajectories of x_1, y_d, and the constraint bounds k_{c_1}, $-k_{c_1}$.

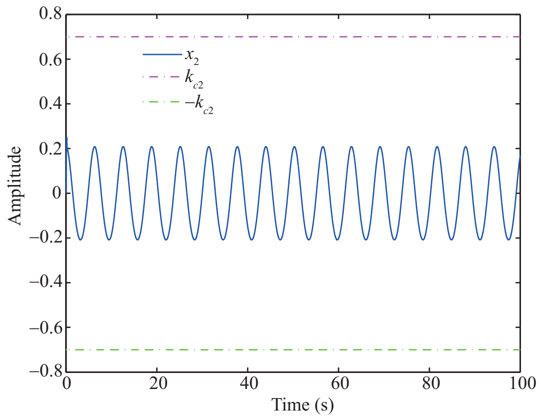

Figure 3 (Color online) The curves of system state x_2 and its constraint bounds k_{c_2} and $-k_{c_2}$.

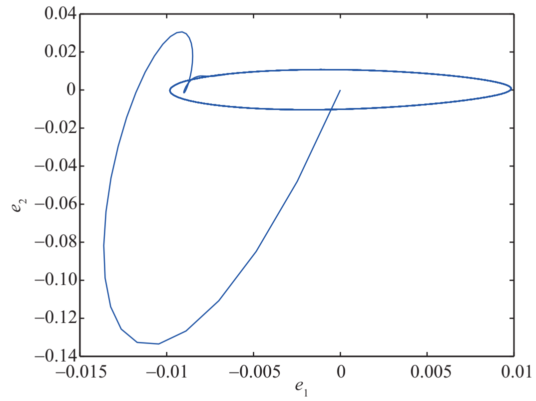

Figure 4 (Color online) The phase diagram of e_1 and e_2.

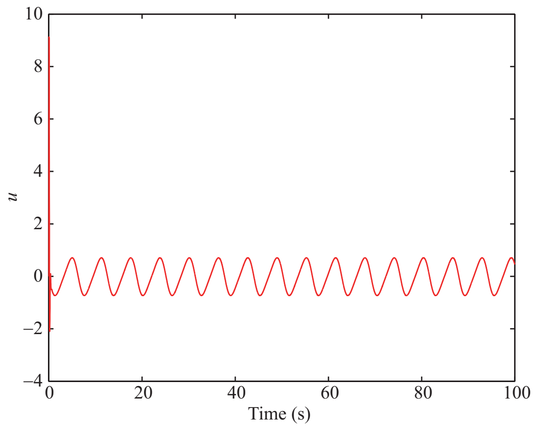

Figure 5 (Color online) The system input u.

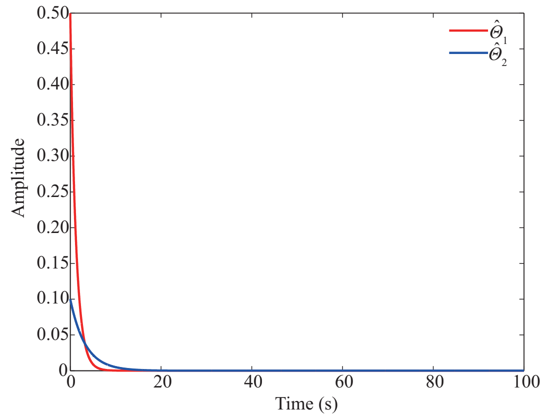

Figure 6 (Color online) The trajectories of adaptive laws $\hat{\Theta}_1$ and $\hat{\Theta}_2$.

5 Conclusion

This paper presents an adaptive control scheme for switched nonlinear systems. The integral Barrier Lyapunov function, a novel type of integral function, is developed to constrain all states within the specified limits. Then, employing Lyapunov stability theory, it is proved that all states do not violate constraint bounds, the errors converge to the small neighborhood of zero, and all signals are bounded. More importantly, the numerical simulation results show the effectiveness of the designed adaptive control strategy. In the future, we hope to address the issue of the explosion of the computation phenomenon of the virtual controller and the actual controller.

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