Ideal Case Study of Adaptive Localization in Storm-scale Ensemble Kalman Filter Assimilation

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Abstract: This study explores the use of the hierarchical ensemble filter to determine the localized influence of observations in the Weather Research and Forecasting ensemble square root filtering (WRF-EnSRF) assimilation system. With error correlations between observations and background field state variables considered, the adaptive localization approach is applied to conduct a series of ideal storm-scale data assimilation experiments using simulated Doppler radar data. Comparisons between adaptive and empirical localization methods are made, and the feasibility of adaptive localization for storm-scale ensemble Kalman filter assimilation is demonstrated. Unlike empirical localization, which relies on prior knowledge of distance between observations and background field, the hierarchical ensemble filter provides continuously updating localization influence weights adaptively. The adaptive scheme improves assimilation quality during rapid storm development and enhances assimilation of reflectivity observations. The characteristics of both the observation type and the storm development stage should be considered when identifying the most appropriate localization method. Ultimately, combining empirical and adaptive methods can optimize assimilation quality.

Key words: EnSRF; storm-scale; hierarchical ensemble filter; adaptive localization

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1 INTRODUCTION

The ensemble Kalman filter (EnKF), developed by Evensen \cite{1} in 1994 based on the extended Kalman filter, was originally introduced to assimilate meteorological and oceanographic data. Accordingly, the ability of EnKF to dynamically estimate the background error covariance is advantageous in theory (Houtekamer and Mitchell\cite{2,3}). Nevertheless, some critical issues restrict the assimilation effect of EnKF, making it necessary to resolve the problems plaguing EnKF in data assimilation applications.

Because computing power is finite and limited, a small ensemble is usually used to estimate the background error covariance. However, the number of ensemble members is typically much lower than the degrees of freedom in a model, producing sampling errors in the statistical ensemble process and introducing spurious correlations between observations and long-distance model state variables. Furthermore, this approach underestimates the background error covariance; as a result, observations are gradually rejected, potentially causing EnKF to fail after several assimilation cycles (that is, causing filter divergence). Ultimately, an optimal analysis may not be achievable, or the entire assimilation may fail. Therefore, constructing a weight function to localize the influence of observations on the state variable is critical for improving the assimilation effect.

Houtekamer and Mitchell \cite{6} were the first to notice the abovementioned spurious sample correlations and proposed a solution, namely, the cutoff radius, which is also known as the original localization method; as such, they defined the relationship between the cutoff radius and the number of ensemble members, where a smaller number corresponds to a smaller cutoff radius and vice versa. Houtekamer and Mitchell \cite{6} used the Schur product theorem to apply a distance correlation function to estimate the ensemble covariance; they further tested their function, and the results demonstrated that the use of covariance localization significantly improves the analysis effect, allowing this method to be widely used. Based on the defined signal-to-noise ratio, Hamill et al. \cite{4} discussed the reasons why covariance localization can improve the ensemble analysis effect. Ott et al. \cite{5} proposed the local ensemble Kalman filter (LEKF), and the assimilation of this scheme was carried out entirely in a local area. Hunt et al. \cite{6} proposed a more computationally efficient local ensemble transform Kalman filter (LETKF) based on LEKF, and Szunyogh et al. \cite{7} subsequently tested the accuracy and computational efficiency of LETKF on
parallel systems using the Global Forecast System produced by the National Centers for Environmental Prediction (NCEP). The LETKF method adopts a slow-change assumption and is therefore suitable for large-scale applications. In 2007, Anderson \cite{8} proposed a localization method for the hierarchical ensemble filter that can automatically adjust the localization distance and weight coefficient generated by changes in the ensemble members; however, flow-dependent features are not considered in other methods. Subsequently, in 2012, Anderson \cite{9} proposed a localization method for correcting the sampling error in a simple model by using the offline Monte Carlo technique to construct a localization function related to the sample correlation coefficient and the ensemble scale.

In a simple model, such as the univariate low-dimensional Lorenz 96 model, the influences of observations can be easily localized. However, in multivariate and multidimensional forecasting models of the atmosphere and ocean, these influences become much more complicated. The localization method of Houtekamer and Mitchell \cite{3,10}, which addresses spurious correlation problems, is widely applied in the construction phase of the Weather Research and Forecasting ensemble square root filtering (WRF-EnSRF) assimilation system \cite{12}. Since this method uses an empirical fifth-order distance correlation function, this approach is referred to as the empirical localization scheme. However, this kind of processing needs to be clarified: the influence of the observation on the model state variable is entirely determined by its distance, and the influence weight decreases as the distance increases, which is not the case. In contrast, the hierarchical ensemble filter is based on the Monte Carlo method, it reflects flow-dependent features, and it can obtain a more reasonable localization function adaptively. Hence, based on the assimilation results of Anderson \cite{8} in a simple model, this paper applies the hierarchical ensemble filter as an adaptive localization scheme to the EnKF assimilation system of the WRF model. The results of a series of ideal experiments involving storm-scale EnKF assimilation using adaptive localization are compared with those of experiments using traditional (empirical) localization, and the application characteristics and assimilation effects of the adaptive localization scheme are analyzed.

2 THE ADAPTIVE LOCALIZATION

2.1 A hierarchical ensemble filter

In 2007, Anderson \cite{8} proposed the hierarchical ensemble filter technique and tested its effects on the simple dynamic Lorenz 96 model. Assume that \( m \) groups of \( n \)-member ensembles (\( m \times n \) total members) are available. When linear regression is used to compute the increment in a state variable, \( x \), given the increments for an observation, \( m \) sample values of the regression coefficient \( \beta \) are available. The regression coefficient for each of the \( i \) groups is calculated the same way as in a standard ensemble filter as \( \beta_i = \sigma_{x,y}/\sigma_{y,y} \), where \( \sigma_{x,y} \) is the a priori sample covariance of the state variable \( x \) and the observed variable \( y \) and \( \sigma_{y,y} \) is the prior variance in the observed variable.

Neglecting other error sources, suppose that the correct but unknown value of the regression coefficient \( \beta \) should be randomly obtained from the same distribution of \( m \) regression coefficient samples. The uncertainty associated with the sample value of \( \beta \) for a given ensemble implies that the increments computed for the state variable are also uncertain. A regression confidence (weighting) factor, \( \alpha \), is defined to minimize the expected root mean square (RMS) difference between the increment in a state variable and the increment that would be used if the correct regression factor were used. \( \alpha \) is chosen to minimize the RMS difference as follows:

\[
\text{RMS} = \sqrt{\frac{m}{\sum_{j=1}^{m} \sum_{i=1}^{m} (\alpha \beta_i - \beta_j)^2}}
\]

This expression equals finding the value of \( \alpha \) that achieves the desired minimization.

\[
\text{RMS}^2 = \sum_{j=1}^{m} \sum_{i=1}^{m} (\alpha \beta_i^2 - 2\alpha \beta_i \beta_j + \beta_j^2)
\]

Taking the derivative with respect to \( \alpha \) and seeking a minimum provides

\[
2\alpha \sum_{j=1}^{m} \sum_{i=1, i \neq j}^{m} \beta_j^2 - 2 \sum_{j=1}^{m} \sum_{i=1, i \neq j}^{m} \beta_i \beta_j = 0
\]

The first sum in Formula (3) can be rewritten as

\[
\alpha \sum_{j=1}^{m} \sum_{i=1, i \neq j}^{m} \beta_i^2 = (m-1) \sum_{j=1}^{m} \beta_j^2
\]

The second sum can be rewritten as

\[
\sum_{j=1}^{m} \sum_{i=1, i \neq j}^{m} \beta_i \beta_j = \left( \sum_{j=1}^{m} \beta_j \right)^2 - \sum_{j=1}^{m} \beta_j^2
\]

Finally, the confidence factor \( \alpha_{\text{min}} \) as the weight coefficient of the observation of a state variable can be computed from formula (6); if \( \alpha_{\text{min}} \) is less than zero, it is set to zero. The set of \( \alpha_{\text{min}} \) for a given observation and the set of model state variables can be viewed as a localization.

\[
\alpha_{\text{min}} = \max \left( \left[ \left( \frac{1}{m} \sum_{j=1}^{m} \beta_j \right)^2 - \frac{1}{m} \beta_j^2 \right] / (m-1), 0 \right)
\]

2.2 Procedure in an assimilation system

EnKF estimates the background error covariance through the statistics of ensemble forecast members. The assimilation method is divided into two steps. The first step is to perform an ensemble prediction; that is, the ensemble of analysis fields at the previous time is taken as the initial value to predict the ensemble of prediction fields at the current time. The second step is to analyze the results of the ensemble prediction, thereby obtaining the ensemble of analysis fields. The formulas are given as follows:
\[ X_i^a(t) = MX_i^a(t-1) \]  
(7)

\[ X_i^b(t) = X_i^b(t) + K \left( y - HX_i^b \right) \]  
(8)

\[ p^b H^T \cong \left[ \left( X_i^b - \overline{X} \right) \left( HX_i^b - \overline{HX}^b \right)^T \right] \]  
(9)

\[ HP^b H^T \cong \left[ \left( HX_i^b - \overline{HX}^b \right) \left( HX_i^b - \overline{HX}^b \right)^T \right] \]  
(10)

\[ K = P^b H^T \left( HP^b H^T + R \right)^{-1} \]  
(11)

where Formula (7) is the ensemble prediction, \( M \) is the prediction model, \( X \) is the state vector of the model prediction, \( a \) is the analysis field, \( b \) is the background field, and \( i \) is the \( i \)th member. Formula (8) represents the ensemble analysis, also called the update equation or the second step, where \( O \) is the observation field, \( H \) is the linear observation operator, \( K \) is the Kalman gain matrix, \( P^b H^T \) represents the background error covariance of the model variables and observed variables obtained by the statistical ensemble members, and \( HP^b H^T \) represents the background error covariance of the observation variables obtained by the statistical ensemble members.

EnSRF is an important branch of EnKF techniques. The traditional EnKF update equation is adjusted to not disturb the observation without underestimating the analysis error covariance, which avoids the sampling error and filtering divergence caused by the disturbed observation. In EnSRF, Formula (8) is divided into two parts: a mean and a disturbance (Whitaker and Hamill[14]). The hierarchical ensemble filter technique is applied to EnSRF, and the formula for calculating each group of regression coefficients \( \beta_i = \sigma_{xi}/\sigma_{yi} \) is given as follows:

\[ \beta_i = P^b H^T / HP^b H^T, i = 1, 2 \cdots m \]  
(12)

For a given observation, the observation and the model state variable are calculated to obtain a series of \( \sigma_{min} \) (Formula (6)). This series of regression confidence factors constitute the localization function, and \( \sigma_{min} \) is the localization weight coefficient (or factor) of the observation for a certain model state variable. Upon introducing the localization function, the analytical equation of EnSRF becomes the following expression:

\[ \overline{X}^a = \overline{X}^b + \sigma_{min} K \left( y - HX^b \right) \]  
(13)

\[ X_i^a = X_i^b - \sigma_{min} \cdot \overline{KHX}_i^a \]  
(14)

where \( \overline{\cdot} \) denotes an ensemble mean and the symbol \( \cdot \) is an ensemble perturbation.

The above method is based on the correlation of the errors between the observations and state variables of the background field to yield different weights of localization influence. These localization weight coefficients are updated whenever an observation is assimilated, which reflects the flow-dependent features. Therefore, the hierarchical ensemble filter is used as an adaptive localization scheme. In addition, the empirical localization scheme of the assimilation system uses the empirical method proposed by Houtekamer and Mitchell[3,10], which uses the Schur product theorem, and a distance-dependent segmented rational function is applied as localization weight coefficients to estimate the ensemble covariance.

3 DATA AND EXPERIMENTAL DESIGN

3.1 Ideal case and simulated observations

For the ideal storm, the WRF’s supercell storm case (a classic super storm that struck central Oklahoma, USA, on 20 May 1977), characterized by rapid movements and constant splitting, is adopted (Skamarock et al.[15]). The true simulation is initialized from a modified real sounding plus a 3 K ellipsoidal thermal bubble centered at \((x, y, z)\) coordinates of \((60, 100, 1.5)\) km with radii of 6 km in the horizontal \((x, y)\) directions and 1.5 km in the vertical \(z\) direction, and the thermal temperature decreases with radius. The sounding corresponds to a strongly unstable atmospheric stratification, as can be seen from the single-point vertical sounding profile (Fig. 1). The thermal temperature stratification curve from 850 hPa to 200 hPa is skewed to the left of the dry adiabatic line, which reflects the instability of the atmosphere; the maximum convective available potential energy reaches 2,714 J/kg, and the stratification curve of the dew-point temperature also appears to be unstable with an upper dry layer and a lower wet layer. Further, the middle and lower wind fields of the troposphere exhibit strong vertical shear, so the environmental field generated by this sounding is rather conducive to the occurrence of strong convective weather.

Simulated radar observations are used in the
experiments, which are calculated based on the true field combined with the radial velocity and reflectivity observation operator. The specific computational equations and parameter settings are the same as those used by Tong and Xue\textsuperscript{16}, and the radar is simulated at the model-level grid coordinates (25, 75). The spatial distributions of the simulated radar reflectivity and radial velocity at 90 min are shown in Fig. 2, revealing that the simulated radar observations are not evenly distributed among all the model grid points. The observations are plotted via trilinear interpolation from the true state grid on polar coordinates according to the WSR-88D radar storm-mode scan configuration, which contains 14 scan elevation angles of 0.48°, 1.45°, 2.4°, 3.3°, 4.3°, 5.2°, 6.2°, 7.5°, 8.7°, 10.0°, 12°, 14.0°, 16.7°, and 19.5°. The corresponding elevation angles are the same as those presented by Xu et al.\textsuperscript{17}. In the horizontal direction, the simulated observations are located at the model grid points, and the horizontal resolution is simply set to be the same as the model grid resolution. The near-real observations of the cone elevation reflect the distribution characteristics of the actual radar observations.

3.2 Experimental design

The WRF prediction model and the EnSRF (Whitaker and Hamill\textsuperscript{14}) assimilation system are utilized for the experiments. The physical domain of the model prediction is 200 km × 200 km × 20 km and is covered by 100 × 100 × 40 grid points with a resolution of 2 and 0.5 km in the horizontal and vertical directions, respectively. The analytic and prognostic variables include the wind components $u$, $v$, and $w$; the disturbing potential height $p_\theta$; the potential temperature $\theta$; and the mixing ratios for water vapor $q_v$, rainwater $q_r$, snow $q_s$, cloud ice $q_i$, and graupel $q_g$.

True storm simulation: the length of the truth simulation is 90 min with an integration step size of 12 s, the 6-phase ice WSM6 microphysics scheme, the rapid radiative transfer model longwave radiation scheme, and the Dudhia shortwave radiation scheme with no cumulus parameterization scheme or land surface process scheme. The lateral boundary conditions are open boundaries, whereas the top and bottom boundaries are rigid boundaries.

Assimilation experiment settings: a 40-member ensemble is used for all assimilation experiments (Xue et al.\textsuperscript{18}). The initial ensemble forecast begins at a model time of 20 min. To initialize the ensemble members, random perturbations sampled from Gaussian distributions with zero mean and standard deviation of 3 m/s for $u$, $v$, and $w$, 3 K for potential temperature $\theta$, and 0.0005 kg/kg for $q_i$ are added. Other model state variables are not perturbed. The assimilation system parameters are selected as follows: covariance relaxation inflation (Zhang et al.\textsuperscript{19}) is used, where the weight of the analysis error covariance is 0.5; the distance correlation function (Gaspari and Cohn\textsuperscript{20}) is used for empirical localization, where the horizontal and vertical localization distances are shown in Table 1; and the observation errors are 0.5 m/s and 2 dBZ for the radial velocity and reflectivity, respectively.

The assimilation experiments are divided into two groups (Table 1). The first group includes tests 1–3, which assimilate only the radial velocity data, to test the feasibility of the hierarchical ensemble filter, and the assimilation effects. The second group includes tests 4–7, which assimilate both radial velocity and reflectivity data. The effects of empirical and adaptive localizations are compared where the observation information is sufficiently comprehensive.

Test 1: Empirical localization experiment: the initial field is not added to the triggering convective thermal bubble, while the other settings are the same as those in true storm simulation. The ensemble forecasts are conducted until 25 min to start assimilating the radial velocity data, and observations are assimilated every 5 min and end at 90 min.

Test 2: Adaptive localization experiment: the adaptive localization with the hierarchical ensemble filter is applied, while the other settings are the same as in Test 1. In consideration of practical calculations, localization distances are adopted and set at the same values as those in Test 1.

Test 3: Sensitivity experiments for the distance scales of the empirical and adaptive localizations (see Table 1).

Test 4: Radial velocity and reflectivity data are assimilated simultaneously, and the other settings are the same as those in Test 1.

![Figure 2](image-url)

**Figure 2.** Three-dimensional distributions of the simulated Doppler radar observations at 90 min: (a) radar reflectivity (dBZ) and (b) radial velocity (m/s).
Test 5: Radial velocity and reflectivity data are assimilated simultaneously, and the other settings are the same as in Test 2.

Test 6: Radar radial velocity and reflectivity data are assimilated simultaneously, and the settings are the same as in Test 2 before 70 min and Test 1 after 70 min. This scheme is referred to as hybrid scheme 1.

Test 7: Radar radial velocity and reflectivity data are assimilated simultaneously. The empirical localization is adopted to assimilate the radial velocity data, while the adaptive localization is adopted to assimilate the reflectivity data. The other settings are the same as in Test 1. This scheme is referred to as hybrid scheme 2.

For tests 2–3, there are 8 groups with 5 members in each group. For tests 5–7, there are 10 groups with 4 members in each group.

Table 1. List of experiments for different localization schemes and localization radii.

<table>
<thead>
<tr>
<th>Experiment Name</th>
<th>Localization</th>
<th>Radial Velocity</th>
<th>Reflectivity</th>
<th>Potential Temperature/Wind/</th>
<th>Disturbing Potential Height</th>
<th>Hydrometeors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>Empirical scheme</td>
<td>Empirical scheme</td>
<td>—</td>
<td>6 km</td>
<td>2 km</td>
<td>4 km</td>
</tr>
<tr>
<td>Test 2</td>
<td>Adaptive scheme</td>
<td>Adaptive scheme</td>
<td>—</td>
<td>6 km</td>
<td>2 km</td>
<td>4 km</td>
</tr>
<tr>
<td>Test 3</td>
<td>Empirical scheme</td>
<td>Empirical scheme</td>
<td>—</td>
<td>8 km</td>
<td>2 km</td>
<td>6 km</td>
</tr>
<tr>
<td>Test 4</td>
<td>Empirical scheme</td>
<td>Empirical scheme</td>
<td>—</td>
<td>4 km</td>
<td>2 km</td>
<td>2 km</td>
</tr>
<tr>
<td>Test 5</td>
<td>Adaptive scheme</td>
<td>Adaptive scheme</td>
<td>—</td>
<td>4 km</td>
<td>2 km</td>
<td>2 km</td>
</tr>
<tr>
<td>Test 6</td>
<td>Hybrid scheme 1</td>
<td>Both (from 70 min)</td>
<td>Empirical scheme</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Test 7</td>
<td>Hybrid scheme 2</td>
<td>Empirical scheme</td>
<td>Adaptive scheme</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

In Test 2, for both the horizontal and vertical distributions, the localization weight functions have completely different patterns for different state variables (Fig. 3 (d-i)). Generally, this function satisfies the characteristic of a smaller weight coefficient at long distances; however, this feature is not strictly dependent on distance but is instead irregular. It is worth noting that the distance scales vary among different state variables; even between hydrometeor variables, such differences are still significant. The structure of the localization function may exhibit multiple peaks (Fig. 4 (c, d and f)), and the distributions appear to display non-Gaussian characteristics. In addition, the patterns of the adaptive localization functions are totally different for different numbers of assimilation cycles and at different observation locations (figure omitted); these features are clearly different from those of the empirical localization functions.

For Test 5, where the simulated radar observations of both the radial velocity and the reflectivity are assimilated simultaneously, Fig. 5 presents the horizontal and vertical distributions of the localization weight coefficients, and Fig. 6 displays the structures of the localization functions. The patterns of the weight coefficients for the radial velocity (Fig. 5 (a-f)) and the reflectivity (Fig. 5 (g-l)) are completely different, even at the same location and for the same state variable. Comparing Fig. 5 (a-f) with Fig. 3 (d-i) further reveals obvious differences in assimilating the same radial velocity at the same location but with different numbers of groups and experimental settings; additionally, the structures of the localization function (Fig. 6 (a-f)) show consistent characteristics. In contrast, in tests 4, 6 and 7, once the empirical localization scheme is adopted, the localization weight function remains constant regardless of how the experimental settings change.

The above analysis demonstrates that the hierarchical ensemble filter adjusts the appropriate scheme to localize
Figure 3. Horizontal (a-f) and vertical distributions (g-i) of the localization weight coefficients for a simulated radar observation at (43, 40, 25) at 60 min for (a-c) Test 1 (contours; values greater than 0.005) and (d-i) Test 2 (shaded, values greater than 0.005; contours, values of 0.1 and 0.5): (a, d, g) $v$-wind; (b, e, h) $w$-wind; and (c, f, i) water vapor mixing ratio ($q_v$).

Figure 4. The localization weight coefficients for a simulated radar observation at (43, 40, 25) on cross sections along $y=40$ and $z=25$ (a, c), $x=43$ and $z=25$ (b, d), $x=43$ and $y=40$ (e, f) at 60 min for (a, b, e) Test 1 and (c, d, f) Test 2.
the influence of an observation on a state variable according to the ensembles and adapts to different numbers of assimilation cycles, different observations and locations, and different state variables, among other factors. This adaptive localization scheme is a complicated function of numerous factors, including the observation type, state variable type, spatial locations of both the observations and the state variables, and time; these features reflect the flow-dependent characteristics of the scheme rather than relying on experience. As a consequence, a priori knowledge of the distance is not needed.

4.2 Assimilations using only radial velocity data

4.2.1 VERIFYING THE FEASIBILITY OF ADAPTIVE LOCALIZATION

Here, the sensitivity experiments of adaptive localization for the group size are performed. The assimilation effects for group sizes 4, 5, 8, 10 and 20 are compared for a total of 40 members. Fig. 7 presents the time-averaged root mean square errors (RMSEs) of the ensemble mean. The RMSEs of groups sizes 4, 5 and 20 are the largest, and their assimilation effects are worse than the other groups for all model variables except $q_r$. The impact of group sizes 8 and 10 is better than others, which may be attributable to group size, and the number of ensemble members in each group is relatively moderate when solving for the $\beta_i$ of each group or calculating $\alpha_{\text{min}}$. In addition, Anderson [8] experimentally determined that when the uncertainty is large, small groups cannot distinguish the signal from noise, and the observation cannot influence the state variable. Thus, based on the above results, the group size selected for Test 2 is 8, with 5 ensemble members per group.

The evolutionary characteristics of the true storm are compared in different periods. Before 60 min, the storm is in a stage of rapid development, in which the original cell rapidly splits into multiple separate cells, which is the typical behavior of a supercell storm. After that, the storm structure is relatively stable, featuring linear development and movements (Wang et al. [13]). Fig. 8 presents the
horizontal distributions of the physical variables at 5 km above ground level (AGL) for the true storm and Test 2. By 40 min, the true storm has split, resulting in two large-value centers on the vertical velocity and two circulation centers on the wind fields. The storm in Test 2 appears to split and form horizontal circulations, but the vertical velocity is smaller, and the intensity and range of the convective cloud are weaker than the truth. At 60 min, the basic shape of the storm in Test 2 is similar to the truth, and the intensities and ranges of the horizontal circulation and the updraft centers are improving. But the intensity of the southern convection center is slightly stronger than that of the true storm.

At 90 min, the true storm discernibly splits, and the two updraft centers are 20 km to the southeast and northeast of the storm position at 40 min. A new convection forms at the storm’s position at 40 min, exhibiting very weak. The convective cloud splits into four centers, two of which are relatively strong and located behind the vertical ascending center. The area of hydrometeors and ascending motion do not spatially coincide, which facilitates the maintenance and development of convection in the storm (Zhu et al. [21]). Here, the physical fields in Test 2 are very close to the

Figure 6. The localization weight coefficients for radial velocity (a, c, e) and reflectivity (b, d, f) at (43, 40, 25) on cross sections at y=40 and z=25 (a, b), x=43 and z=25 (c, d), x=43 and y=40 (e, f) at 60 min for Test 5.

Figure 7. Time-averaged RMSEs for different group sizes, where the horizontal axis represents the model variable, (a) the mixing ratios for water vapor ($q_v$), rainwater ($q_r$), snow ($q_s$) and graupel ($q_{gr}$), and (b) wind components $u$, $v$, and $w$. The intensity of the southern convection center is slightly stronger than that of the true storm. At 90 min, the true storm discernibly splits, and the two updraft centers are 20 km to the southeast and northeast of the storm position at 40 min. A new convection forms at the storm’s position at 40 min, exhibiting very weak. The convective cloud splits into four centers, two of which are relatively strong and located behind the vertical ascending center. The area of hydrometeors and ascending motion do not spatially coincide, which facilitates the maintenance and development of convection in the storm (Zhu et al. [21]). Here, the physical fields in Test 2 are very close to the
truth, and much of the error is corrected. Therefore, Test 2 can retrieve the main characteristics of the true storm satisfactorily. However, the new convection generated behind the true storm is not reflected, and some differences with the truth can be detected in the structural details of the northern ascending area and the convective cloud. Furthermore, the results of Test 1 are not much different from those of Test 2 (figure omitted). These results indicate that the adaptive localization using the hierarchical ensemble filter is feasible in the WRF-EnSRF assimilation system for storm-scale EnKF assimilation, and the model storm can be rebuilt remarkably well after a sufficient number of assimilation cycles.

4.2.2 ASSIMILATION EFFECT ANALYSIS

To quantitatively evaluate the assimilation effects of the empirical and adaptive localizations, the RMSEs of the analyzed model state variables are calculated against the truth in the convective region. The calculations are conducted over the grid points with the reflectivity from the truth greater than 10 dBZ, as in previous studies (Tong and Xue [16]; Xu et al. [22]). As shown in Fig. 9, the RMSEs of all variables in Test 1 are lower than those in Test 2, except \( q_r \). During the several initial assimilation cycles, the effects of the two localization schemes are equivalent. However, with increasing assimilation time, the empirical localization gradually becomes superior, especially in the temperature and wind variable analyses. Ultimately, the empirical localization outperforms adaptive localization in the quantitative analysis performance when assimilating only radial velocity data.

Figure 10 presents the physical variable fields in the vertical cross section along \( y=44 \) for the true storm and assimilation tests 1 and 2 at 90 min. The physical variable fields in Test 1 and Test 2 are similar to those of the true storm, especially in the establishment of the main warm center of the convective zone. The extents of this warm center in these two assimilation experiments are nearly comparable to the truth. The inclined ascending motion structures of the storms are similar to the truth, and a 5 K disturbing potential temperature center is simulated in front of the storms. In comparison, the height of the larger graupel mixing ratio in Test 2 is slightly higher than that of the true storm, and the detailed structure of the convection center in Test 2 is worse than that in Test 1. The experimental results reveal that assimilation using adaptive localization can establish the interaction and connection between the ascending motion and latent heating of a convective cell suitably; thus, the storm structure and its features can be simulated reliably in the development stage. However, compared with using empirical localization, the assimilation effect of adaptive localization does not show obvious advantages. The reason may be that the assimilation of radial velocity data mainly reflects the wind field information, and the two localization methods yield the localization weight coefficients of the wind variables with similar distributions. In addition, the influences of wind observations are more strongly correlated with the distance, and the empirical function provides a better description of this distance correlation.

4.2.3 SENSITIVITY OF THE DISTANCE SCALE

The localization distance scale, which defines the influence range of the observation, is also given in the adaptive localization experiments. The influence weight of
the observation on the state variable is adaptively adjusted following a change in the ensemble within the finite physical domain. A series of distance scale sensitivity experiments are conducted to test the influence of the distance scale on the assimilation quality. For the adaptive localization, the RMSEs in Test 2, Test 3-2, and Test 3-4 are relatively close; among them, Test 2 has a smaller error (Fig. 9). The reason may be that the distance scale is too small to appropriately transmit the observation information well. In contrast, if the scale is too large, a spurious correlation between the observation and the background field arises. In general, although the distance scale affects the results, the assimilation is not sensitive to this scale.

The empirical localization experiments also yield similar results, consistent with Lan et al.’s research results [23]. However, the analytical results of the hydrometeor variable differ significantly at the beginning of several assimilation cycles when the distance scale is varied. In summary, the adaptive localization is less sensitive than the empirical localization to the distance scale during the entire assimilation period.

4.3 Assimilations using both radial velocity and reflectivity data

4.3.1 Quantitative analysis of the assimilation effect

The assimilation of radial velocity data improves...
mainly the wind field and the position of the radar’s strong echo band, while the assimilation of reflectivity data can improve the configuration of microphysical and dynamic fields (Liu et al. [24]), making the simulated storm structure closer to the truth. Here, both radial velocity and reflectivity data are assimilated, and the assimilation effects of the empirical and adaptive localization schemes are evaluated quantitatively by using the ensemble mean RMSEs in tests 4–7 (Fig. 11). Up to 70 min, the assimilation effect in Test 5 using the adaptive localization is significantly better than that using the empirical scheme in Test 4 for all the variables, especially for the wind and temperature variables. The assimilation effect of the empirical localization is better in the final 10–20 min, and the RMSEs in Test 4 are smaller than those in Test 5 (especially the wind variables) at the end of the assimilation. When the storm is changing mainly in a nonlinear fashion during the rapid development stage, the assimilation effect of the adaptive localization is better; in contrast, when the storm structure is relatively stable and dominated by linear development and linear movement, the empirical localization has more advantages. It may be relevant that the influence of observation in the empirical localization decreases monotonically with increasing distance, and its weight coefficients are priori determined values, while the adaptive localization adaptively determines the weight coefficients through the ensemble, which is strongly nonlinear in principle and coincides more with the variation characteristics of the rapidly developing storm, especially for hydrometeor variables.

Test 6 (hybrid scheme 1) is designed based on the above analysis. Specifically, the adaptive localization is adopted from the onset of assimilation to 70 min, and then, the empirical localization is implemented until the end of the assimilation. The RMSEs in Test 6 are lower than those in Test 5, similar to that in Test 4 in the last 20 min, so the assimilation effect of Test 6 is improved (Fig. 11). Compared with the adaptive scheme, the empirical scheme exhibits slightly better quality when only the radial velocity is assimilated, whereas the adaptive scheme improves the assimilation effect when the reflectivity is added in the assimilation. Therefore, Test 7 (hybrid scheme 2) is designed as follows: the empirical localization is applied to assimilate the radial velocity, and the adaptive localization is used to assimilate the reflectivity. When comparing the effect of all the assimilation cycles (Fig. 11), the RMSEs in Test 7 are smaller than those in Test 4. Furthermore, acceptable results are maintained in the later assimilation cycles in Test 7, and the RMSEs converge gradually, unlike in Test

![Figure 11](image-url). The evolution of the ensemble mean RMSEs in the convection area for Test 4 (gray dashed-dotted lines), Test 5 (dotted lines), Test 6 (dashed lines), and Test 7 (solid lines). Units are shown in the plots.
5. Additionally, the RMSEs in Test 7 are slightly smaller than those in Test 6 at the end of the assimilation. In conclusion, the RMSEs of hybrid scheme 2 are the smallest, and the assimilation effect of this scheme is the best throughout the entire assimilation period. The reason may be that the radial velocity and reflectivity reflect different types of main information: the radial velocity reflects mainly the information of the wind field, while the reflectivity is more directly related to the hydrometeors and is essential for retrieving the microphysical fields. The influences of wind observations may conform to the distance correlation, as reflected by the empirical function, but the same does not hold for the influences of the hydrometeor variables, which are more complex and irregular. Note that the reflectivity observation operator (Tong and Xue\cite{16}) is nonlinear, and it carries more uncertainties than the radial velocity operator. Furthermore, the adaptive scheme applies a flow-dependent adjustment to the influence of an observation based on the error covariance of the state variables, which is more suitable. Therefore, hybrid localization is indicated to be a more reasonable choice when adding reflectivity data to the assimilation.

Figure 12 presents the vertical distributions of the horizontal spatial mean RMSEs of the storm convective zone at the end of the assimilation. The assimilation performance of Test 5 appears to be poorer than its counterparts, particularly regarding the wind and pressure variables and the graupel mixing ratio, which are consistent with that shown in Fig. 11. There is almost no difference in the assimilation effect of the rainwater mixing ratio among tests 4–7. Excluding the individual hydrometeor variables, the maximum error in each state variable is found at the upper level, and likewise, and the differences among each test are also most significant at the upper level, whereas the discrepancies in the middle and lower layers are relatively small. Although the RMSEs in Test 6 are reduced in the last few assimilation cycles (Fig. 11), the upper-level errors are slightly larger than that in both Test 4 and Test 7. In Test 7, the error of almost every state variable emerges as the smallest, especially for the wind and temperature variables.

4.3.2 ASSIMILATION EFFECT OF STORM STRUCTURE

The reductions in the assimilation errors for the wind and temperature fields correspond to the improvements in the upper-level divergence, the convective development height, and the thermal-dynamic configuration in the convective zone. The physical variable fields in the vertical cross section within the convective zone of the

Figure 12. The profiles of the ensemble mean RMSEs in the convection area for Test 4 (gray dashed and dotted line), Test 5 (dotted line), Test 6 (dashed line), and Test 7 (solid line), where the vertical axis represents height (km). Units are shown in the plots.
storm in tests 4 and 7 are compared at 40 min, 60 min, and 90 min (Fig. 13). The true storm develops deep convection by 40 min, reaching a maximum height close to 15 km, as well as a secondary vertical circulation with ascending motion in the rear and descending motion in the front of convection. The ascending structure is inclined, facilitating the development of convection. Strong and inclined updrafts can lift larger hail particles and transport them forward at a high altitude, guaranteeing that the falling drag effect of the hydrometeors does not affect the ascending motion behind it. A 5 K warm area forms in the rear of the convective zone with a strong ascending motion and extends from the lower level to approximately 10 km; this structure indicates that a positive feedback configuration between the latent heating and ascending motion in the storm is obvious; the storm depends on this mechanism for development. The storm develops rapidly at 60 min, exhibiting a strong ascending motion and an increase in the area of the larger graupel mixing ratio. The disturbing potential temperature in the convective zone reaches up to 10 K, with the 5 K warm center appearing before the storm. At 90 min, the storm is maintained and dominated by stable movements, with the development slowing down.

At 40 min, the maximum convection height in Test 7 is close to the truth. The warm area and the inclined updraft are also established by this time, with the position and range of the warm area and the ascending motion structure being similar to those of the true storm, although the vertical extent of the warm area is slightly more restricted. The vertical extent of the large-value area of the graupel mixing ratio is similar to the truth, but the range and maximum intensity of this area are smaller. The falling drag and evaporative endothermic cooling effects of the hydrometeors produced by the assimilation may impact the initial development of convection; hence, a gap in the intensity remains between the storm in Test 7 and the true storm. Although the warm area is obtained in Test 4, the structure of this area is distorted, and the vertical extent, position, and range of the large-value area of the graupel mixing ratio are quite different from the truth. Test 7 shows advantages at 40 min regarding the performance of the positive feedback between the latent heat and ascending motion. With increased assimilation cycles, each physical variable field in Test 7 at 60 min moves closer to the truth. In contrast, in Test 4, the height of the warm center of the storm is obviously low, and its shape varies substantially from the true storm; moreover, the newly generated warm area in front of the storm is highly elevated and too broad. At 90 min, the physical variable fields in tests 4 and 7 are similar to those of the true storm. The maximum values of the warm center are slightly lower in both tests, but both can reach 8 K, which is near the truth of 10 K. Compared with those in Test 4 and in the tests where only the radial velocity is assimilated (Fig. 10b, c), the morphology of the warm area and wind field, and the structure, center range and extension height of the graupel mixing ratio in Test 7 are closer to the truth. Therefore, on the basis of the storm development mechanism reflected by the physical variable fields, the

Figure 13. Vertical cross sections of graupel mixing ratio (shaded; kg/kg), disturbing potential temperature (contour; K) and wind (vector; m/s) along $y=51$ at 40 min (a, d, g), $y=49$ at 60 min (b, c, h), and $y=44$ at 90 min (c, f, i) for (a-c) the true storm, (d-f) Test 4, and (g-i) Test 7.
thermal and dynamic configurations, and the storm structure characteristics. Test 7 (hybrid scheme 2) improves the assimilation effect significantly during the entire assimilation period.

5 CONCLUSIONS AND DISCUSSION

As an adaptive localization scheme, the hierarchical ensemble filter was applied to the WRF-EnSRF assimilation system. A series of ideal experiments in storm-scale assimilations were conducted, taking the WRF’s supercell storm case as an example, and the results were compared with those obtained by using the empirical localization scheme. Furthermore, the application characteristics and assimilation effects of the adaptive localization in storm-scale EnKF assimilation were analyzed. The main conclusions can be drawn as follows:

1. The hierarchical ensemble filter provides continuous updating localization influence weights adaptively according to the correlation of the error between the observation and state variable of the background field, which differs from the empirical localization that depends on prior knowledge of the distance between the observation and background field. Adaptive localization using the hierarchical ensemble filter was applied in ideal experiments in which only radial velocity data were assimilated. With increasing assimilation time, the storm simulated by assimilation approached the true storm, verifying the capability and feasibility of the adaptive localization scheme for storm-scale Kalman filter assimilation.

2. When only the radial velocity (reflecting mainly the information of the wind field) was assimilated, the assimilation effect of the adaptive scheme did not show obvious advantages. However, when the reactivity (reflecting mainly the information of hydrometeors) was further assimilated, the adaptive localization scheme was used to assimilate the reactivity, while the empirical localization scheme was used to assimilate the radial velocity; that is, the corresponding localization method was selected according to different characteristics of the observation, which significantly improved the assimilation effect during the whole period.

3. When the storm structure was relatively stable and dominated by linear development and linear movements, the empirical localization scheme with a distance correlation and Gaussian-like features achieved superior performance. When the storm was developing rapidly and nonlinearly, the assimilation effect of the adaptive localization scheme with non-Gaussian features was superior to that of the empirical localization scheme. Therefore, the rational selection of localization schemes for different stages of storm development based on the storm’s characteristics is conducive to optimizing the assimilation quality.

4. The experiments were carried out under the assumption that the model is error-free and the effect of the model itself is neglected. However, the advantages of the adaptability of the proposed scheme originate from the ensemble obtained by integrating the model. Therefore, whether the model is accurate greatly affects whether the localization method is reasonable. In addition, further research should be conducted to ascertain whether the adaptive scheme can improve the quality of EnKF assimilation in assimilation scenarios involving multiple types of data (such as in real cases that include many complicated factors).

REFERENCES


